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Guidelines for componentbased analytical vulnerability assessment of buildings and nonstructural elements

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Guidelines for component-based analytical vulnerability assessment of buildings and nonstructural elements

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A procedure is offered for the analytical derivation of the seismic vulnerability of building classes, that is, probabilistic relationships between shaking and repair cost as a fraction of replacement cost new for a category of buildings. It simulates structural response, damage, and repair cost for the structural and non-structural components that contribute most to construction cost, and then scales up results to account for the components that were not simulated. It does so for a carefully selected sample of building specimens called index buildings whose designs span the domain of up to three features that are believed to most strongly influence seismic vulnerability within the building class. One uses moment matching to combine results for the index buildings to estimate behaviour and variability of the building class. One can simulate non-structural vulnerability alone by ignoring damage and repair cost for structural components. The work is written for a structural engineer with a master's degree, skilled in structural analysis, but not necessarily experienced in loss modelling.

The procedure has five steps. In Step 1, the analyst defines the asset class with one, three, or seven specimens of the asset class; the specimens are called index buildings. The choice depends on available resources and the rigor with which the analyst wants to address variabilities within the building class and within the performance of an individual index building. Each index building is assigned a particular structural and non-structural design, including number of stories, structural material, lateral load resisting system (LLRS), geometry, and quantities of each of the top 1 or 2 structural component categories and top 5 or 6 non-structural component categories.

Step 2 is to derive story-level vulnerability functions, without considering collapse. (Collapse is addressed in a later step.) The vulnerability functions express the repair cost of components on the story as a function of story-level excitation (drift, acceleration, or other measures of story-level structural response). Step 3 is to perform a structural analysis at each of many levels of ground motion with the objective of estimating story-level excitation and collapse probability as a function of ground motion. We offer three options for structural analysis, from a very simple approach to multiple nonlinear dynamic structural analyses; the analyst is free to choose among these, again considering available resources and desired rigor.

Step 4 is to derive a building-level vulnerability function by summing story-level losses over stories, factoring up to account for the fact that only the top 6 to 8 structural and non-structural component categories are inventoried, applying the theorem of total probability to consider the probability of collapse. By omitting the top 2 or so structural components, one can create vulnerability functions for only the non-structural components. The vulnerability function is normalized by replacement cost new to depict damage factor as a function of ground motion.

In Step 5, the mean vulnerability function and coefficient of variation of damage factor for the asset class are calculated. The mean damage factor for the asset class is calculated as a weighted average of those of the

index buildings. The coefficient of variation is calculated by one of three means: using a proxy from HAZUS in the case of a single index building, as a multiple of the variability of vulnerability between index buildings in the case of three index buildings, or in the case of seven index buildings, by calculating the variance of vulnerability of the weighted sample of index-building-level vulnerability functions, including both betweenand within-building variability.

Keywords

Analytical; vulnerability; moment matching; component-based

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GLOSSARY AND ABBREVIATIONS

Most variables are defined near their first use. Other common abbreviations in this document are as follows.

ASCE	American Society of Civil Engineers
ATC	Applied Technology Council
CDF	Cumulative distribution function. Applied to uncertain quantities, gives the probability that the variable will take on a value less than or equal to a particular value, as a function of that particular value.
CUREE	Consortium of Universities for Research in Earthquake Engineering
DF	Damage factor, typically uncertain repair cost divided by replacement cost new.
DP	Demand parameter, a measure of structural response that can be recorded or estimated from the results of a structural analysis. Typical choices are the peak floor acceleration (PFA) and the peak transient interstory drift ratio (PTD), both of which apply to a particular floor (in the case of PFA) or story (in the case of PTD).
E[·]	Expected (mean) value of the quantity in brackets
E.g.	For example
FEMA	Federal Emergency Management Agency
f _H	High fatality rate conditioned on collapse, fraction of building occupants at the time of the earthquake
f _L	Low fatality rate conditioned on collapse, fraction of building occupants at the time of the earthquake
Floor	A floor or roof diaphragm, including the ground floor
Fragility functior	A deterministic relationship between the occurrence probability of some undesirable event and a measure of environmental excitation that is causally related to the event. As used in a

ragility function. A deterministic relationship between the occurrence probability of some undesirable event and a measure of environmental excitation that is causally related to the event. As used in a component-based vulnerability method, it generally refers to the probability that a particular damage state occurs or is exceeded in a building component as a function of the value of the demand parameter to which the component is most sensitive. In a wholebuilding vulnerability method, it may refer to the occurrence probability of some wholebuilding damage state, such as collapse, as a function of ground motion intensity. In either case the fragility function can be interpreted as the cumulative distribution function of the capacity of the component or building to resist the specified damage state, measured in terms of the demand parameter or intensity measure related to the damage state.

GEM Global earthquake model

Geomean Geometric mean

GMPE Ground motion prediction equation, a subclass of which is the attenuation relationship, which relates an intensity measure to earthquake source and path parameters such as magnitude, distance, and site conditions, among others.

HAZUS-MH	Hazards United States Multihazards (a software product of the US Federal Emergency Management Agency; see e.g., NIBS and FEMA 2012)
Highrise	As used here, refers to a building that has 8 or more stories above grade.
ID	Identifier
l.e.	That is
IM	Intensity measure. As used here, IM refers to a quantity that characterizes environmental excitation to the asset. This work deals solely with earthquake shaking, so IM is limited here to a measure of degree of ground shaking, and more particularly to a scalar instrumental measure.
Index building	A particular member of an asset class, with specified geometry, materials, material properties, structural and nonst ructural components, replacement cost, and other features that would appear on a complete set of construction or as-built documents, include structural, architectural, mechanical, electrical, plumbing drawings and specifications.
IT	Information technology
Joint distributio	on The probability density that two or more uncertainties will take on a particular set of values, one for each uncertainty
km	kilometers
Log std deviatio	on Logarithmic standard deviation, that is, the standard deviation of the natural logarithm of the variable
Loss	A scalar nonnegative measure of repair costs, life-safety impacts, or loss of functionality ("dollars, deaths, or downtime").
Lowrise	As used here, refers to a building that has 1 to 3 stories above grade.
MDF	Mean damage factor, the expected value of repair cost as a fraction of replacement cost new.
MDOF	Multi-degree of freedom
MECE	Mutually exclusive and collectively exhaustive
Midrise	As used here, refers to a building that has 4 to 7 stories above grade.
NISTIR	National Institute of Standards and Technology Interagency Report
Param	Parameter
P _c	Probability of collapse
PFA	Peak floor acceleration, generally geometric mean of two orthogonal directions, units of gravity
PMF	Probability mass function
Population	All members of a class, as in all the buildings that belong to a specified asset class.
PTD	Peak transient interstory drift ratio, unitless
RCN	Replacement cost new, that is, the cost to build a replacement for the building on the same site, excluding demolition of an existing building on the site
Ref	Reference, generally meaning a citation for a source of information
Sample	One or more members of a population

SDOFSingle degree of freedomStoryThe space between two floorsUncertaintyAs used here, a variable with an uncertain value, often called a random variable.USUnited States

Vulnerability As used here, a probabilistic relationship between uncertain degree of loss to a specified component, story, particular building, or asset class as a function of environmental excitation. For a component or story, the environmental excitation generally refers here to a story-level structural response measure such as peak transient interstory drift or peak floor acceleration. For a particular building or asset class, the environmental excitation generally refers here to ground-motion intensity such as 5% damped elastic spectral acceleration response at some index period of vibration. When applied to repair cost for a building or asset class, loss is typically normalized by replacement cost new, and is referred to as damage factor. When applied to fatality rate for a building or asset class, the number of fatalities is normalized by the total number of building occupants at the time of the earthquake.

1 Introduction

1.1 Objectives of the methodology

A procedure is offered for the analytical derivation of the seismic vulnerability of buildings by a componentbased approach. By "component-based" we mean a method that estimates damage and loss at the level of individual building components—partitions, ceilings, columns, etc.—and then aggregates to the level of individual buildings, and then aggregates further to a building class. One alternative is to estimate the seismic vulnerability of buildings by a whole-building approach, wherein one estimates overall damage to the building directly, without consideration of each beam, column, wall, or non-structural component. See D'Ayala et al. (2015) for guidelines of how to derive vulnerability functions by a whole-building approach.

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By "seismic vulnerability" is meant a probabilistic estimate of repair cost (as a fraction of replacement cost new) as a function of intensity measure level for an asset class, where "asset class" is generally limited here to categories of buildings, as opposed to bridges, pipelines, sailboats, etc. These guidelines do *not* use "vulnerability" and "fragility" interchangeably. As used here, fragility means the probability of an undesirable outcome occurring given some environmental excitation. These guidelines use the terms "vulnerability function," but not "vulnerability curve" and "fragility curve." The choice is stylistic.

The asset class is defined in whatever terms the analyst has selected from among the GEM building taxonomy version 2.0 (Brzev et al. 2013). For illustration purposes, it is assumed here that the asset class is defined in terms of structural material, lateral load resisting system, height category, occupancy class, and era of construction. These guidelines were originally developed for highrise buildings, but are believed to be applicable to most kinds of mid- and lowrise buildings as well. These guidelines are flexible: they accommodate simple structural analysis that idealizes the building as a single-degree-of-freedom nonlinear oscillator. They also allow for more-complex methods that can account for higher modes and multiple degrees of freedom.

The principal constraint of the methods presented here is the availability of component fragility functions. (Component fragility functions are a subclass of fragility functions, expressing the probability of reaching or exceeding a limit state for a component as a function of demand. See the glossary for more detail.) As of this writing, the principal gaps in available component fragility functions are those for unreinforced walls constructed of brick, concrete block, clay tile, earth, or stone. Until those fragility functions are created, the analyst interested in deriving building-level fragility functions for unreinforced masonry bearing wall buildings and confined masonry buildings is referred to the companion work by D'Ayala et al. (2015).

Earthquake-induced property losses in buildings are often considered to be composed of three components: structural, non-structural, and contents. The division between these groups can sometimes be ambiguous. (E.g., gypsum wallboard partitions in woodframe buildings can act as structural or nonstructural components;

computers in server racks that are permanently fixed to the floor can be seen as nonstructural elements or as contents). This work proposes procedures for developing analytical seismic vulnerability functions for structural and nonstructural components. We will not attempt to draw a crisp distinction between structural, non-structural, and contents, other than a practical one. If a component contributes significantly to the strength and stiffness of the building, it is structural. If it does not contribute significantly to the strength or stiffness of the building, but is typically delivered by the general contractor with new construction or retrofit, it is considered nonstructural. This is important: components that contribute significantly to strength or stiffness but that are not included in the structural analysis are *not* nonstructural; rather their omission from the structural analysis is considered to be an error in modelling on the part of the analyst. Thus, masonry infill walls are structural, even if not considered in the structural design, since they contribute to stiffness. Some component categories may act as structural components in one context and as nonstructural in a nother, for example gypsum wallboard partitions on metal studs may be structural in a gas station, nonstructural in a highrise building.

Finally, some components can be considered contents. For some components such as freestanding bookcases or desktop computers the distinction is clear, but for others it can be problematic to clearly define them as contents as distinct from non-structural components. One can try to use time of installation, whether the component is fixed to the building, and whether the component is ever moved. None of these distinctions seems capable of providing a completely satisfying definition. For example, some refrigerators are built into the building, some are freestanding. Some are installed before the completion of construction, some are installed afterwards. Some are secured to the building structure and rarely if ever move, and some are freestanding on levellers or casters. We leave the distinction to the analyst, but suggest that if a component does not contribute significantly to strength or stiffness, is added by the owner or a vendor after construction is completed, and can be readily moved, it can be considered contents.

The method presented here estimates vulnerability based on the contribution to repair cost from only the top few components. By "top components" is meant the ones that contribute most to construction cost (new). Fragility functions for each category of component are taken from FEMA P-58 (Applied Technology Council 2012). FEMA provides a database of the component fragility functions; see the URL in the referencescited section of this report for instructions on how to acquire the database. Alternatively, the analyst can develop new component fragility functions using procedures specified for FEMA P-58. Fragility functions are defined in terms of the repairs required to restore the component to its pre-earthquake state, rather than in general qualitative terms. Repair costs for each component fragility function are taken from FEMA P-58 with an adjustment to account for local labor costs; the adjustment procedure is provided here. This is important: the FEMA P-58 material provides US-centric component repair costs, but we provide guidance on how to use those repair costs to estimate component repair costs in other countries, using a factor that relates local construction labor costs to US labor costs. And the analyst is free to consult with a local construction contractor to estimate local repair costs for the top components and their damage states.

Our purpose here is to offer a procedure for estimating the vulnerability of a whole aggregate class buildings, meaning all the structural and nonstructural components in a building as a whole, not every doorknob and

exit sign. We also mean a probabilistic relationship between ground motion and repair cost as a fraction of replacement cost new of the entire building (not including contents).

The relationship is defined for an asset class that the user specifies using the GEM building taxonomy version 2.0 (Brzev et al. 2013). Even the basic GEM building taxonomy is very flexible, with effectively unlimited combinations available. For illustration purposes, we assume that the asset class is defined by three attributes: an occupancy class (residential, commercial, or industrial); a height range (low-, mid-, or highrise); and a general category of lateral load resisting system (or more precisely a mode shape representing either an idealized frame, shearwall, or an intermediate case). The height range and lateral load resisting system can alternatively be replaced by calculated mode shape and 3-dimensionsal relationship between 5% damped elastic spectral acceleration response, roof absolute acceleration, and roof relative displacement (relative to the ground). For modern code-conforming structures, we expect that the lateral load resisting system will not make much difference, and that the vulnerability function will only depend on occupancy class and height range.

The methodology can be readily applied to finer occupancy categories—IT manufacturing facilities, for example—or to different combinations of taxonomic features, but we will not elaborate here. The adaption should be fairly obvious. In developing these guidelines, we have striven to achieve the following additional objectives:

- General applicability anywhere in the world.
- Requiring no greater skillset than a structural engineer with a master's degree. The analyst is therefore assumed to be skilled in creating a structural model and in performing structural analysis.
- Allowing for exercise of engineering judgment, but with defaults as guidance for less experienced analysts.
- Brevity (procedure defined in fewer than 50 pages).
- Flexibility (ability to accommodate various levels of effort).

1.2 Brief summary of the methodology

The methodology has five steps, summarized here. Details and justification for these steps are provided in the Section 3.

In Step 1, the analyst defines the asset class with one, three, or seven index buildings. An index building is a single particular specimen building of the asset class, with given geometry, materials, material properties, etc. If one index building is used, the analyst defines the asset class using a typical-quality case.

If the analyst wishes to explicitly quantify uncertainty with modest effort, then three index buildings are defined: a poor, typical, and superior-quality case. The poor-quality index building might be one with relatively low design base shear and relatively fragile or poorly anchored component types. The typical-quality index building might be one with typical (for the class) design base shear and a typical mixture of fragile and rugged component types. The superior-quality index building might be one with a high (for the class) design base shear and a relatively rugged mixture of components.

If the analyst wishes to quantify uncertainty to a high degree, including within-building uncertainty, then seven index buildings are defined that span at least three important dimensions that vary within the asset class. For example, for an asset class defined in terms of structural material, lateral load resisting system, height range, and occupancy, the top three uncertainties that vary within the class might be number of stories, vertical irregularity, and design base shear. The analyst is free to select different top uncertainties if the asset class fixes one or more of these.

Regardless of whether 1, 3, or 7 index buildings are used, each index building is assigned a particular number of stories, structural material per the GEM building taxonomy, lateral load resisting system (LLRS) per the GEM building taxonomy, a broad category of LLRS (shearwall, frame, or mixed), degree of plan irregularity (maximum ratio of long building wing length to shorter wing length), degree of vertical irregularity (maximum ratio of story height below to story height above), design base shear, and lastly, story by story, the quantity (in terms of replacement cost) of each of the top 1 or 2 structural component categories and top 5 or 6 nonstructural component categories. Omit the structural components if the analyst only wants to estimate non-structural vulnerability. Include number of occupants if analyst wishes to estimate fatality risk.

For the case of seven index buildings, three quality-level variants are defined for each index building. The variants differ in the selection of detailed component types. Since structural component types are largely defined by the index building, it is the detailed taxonomic categories of the nonstructural components that vary between variants. To explain, for each nonstructural component category defined at the level of the NISTIR 6389 component taxonomy (from National Institute of Standards and Technology 1999, e.g., C1011 Fixed Partitions), there may be many choices for detailed component category, as defined at the level of FEMA P-58's component taxonomy. For example, FEMA P-58 (Applied Technology Council 2012) currently offers fragility functions and repair costs for 12 types of partitions, such as full-height and partial height gypsum board on metal stud, with wallpaper, ceramic tile, or stone finish. Some of these are relatively fragile, with low median drift capacity, some relatively rugged with median drift capacity 3 times that of the most fragile ones.

In the poor-quality variant, nonstructural components are selected to reflect relatively vulnerable component types, such as unanchored equipment or relatively fragile partitions. In the typical-quality variant, nonstructural components are selected to reflect a typical mix of rugged and more-fragile component types. In the superior-quality variant, nonstructural components are selected to reflect a relatively rugged mix of component types, such as mostly well anchored equipment and relatively rugged structural components.

Step 2 is to derive two story-level vulnerability functions given non-collapse. One of the two is for driftsensitive components and the other for acceleration-sensitive components. (The probability of collapse and the cost given collapse is addressed in a later step.) The vulnerability function is essentially the sum over all the components of the product of the probability of the component being in each damage state, the repair cost per unit of the component given the damage state, and the quantity of the components at that story. The story-level vulnerability function gives the mean damage factor of the components on that story, as a function of the peak drift or peak floor acceleration at that floor, given non-collapse. Note that, if analyst only wants to estimate fatalities, he or she can skip this step.

Step 3 is to perform a structural analysis with the objective of estimating peak transient drift ratio at each story, peak floor acceleration at each floor, and collapse probability. All three measures (drift by story, acceleration by floor, and collapse probability) are evaluated at each of many intensity measure levels. The analyst is free to choose the intensity measure type of interest, but we offer default intensity measure types: for one index building, we suggest choosing the geometric-mean 5% damped elastic spectral acceleration response at 0.3 second period for lowrise (1-3 story) construction or 1.0 second period for taller buildings. For three or seven index buildings, we recommend the geometric mean of the 5% damped elastic spectral acceleration acceleration response at the estimated fundamental period of vibration of each building.

We offer three options for structural analysis. The first assumes a mode shape appropriate either to a shearwall system, a frame system, or a mixed system. If shearwall system, the mode shape is calculated by first principles from the deformed shape of a prismatic cantilever column with infinite shear stiffness and finite bending stiffness and subjected to a triangular distributed lateral loading pattern. If a frame system, the mode shape is calculated by first principles from the deformed shape of a prismatic cantilever column with finite shear stiffness and infinite bending stiffness and subjected to a triangular loading pattern. If a mixed system, the model shape is triangular. Peak transient drifts and floor accelerations are then calculated based on the mode shape and spectral acceleration response of an equivalent single-degree-of-freedom harmonic oscillator with an elastic-perfectly plastic pushover curve. The second option is to use nonlinear pseudostatic (pushover) analysis. The third structural analysis option is to perform multiple nonlinear dynamic structural analyses of each index building. We offer guidance on selecting ground motions and performing structural analyses from FEMA P-58. A fourth structural analysis option is for the analyst to perform any type of structural analysis that is convenient, familiar, and deemed professionally sound by the project team, as long as (1) it estimates the structural response measures of interest at each of the intensity measure levels of interest, and (2) the structural analysis procedures—their strengths and weaknesses and the reasons the analyst selected them-are disclosed to the consumer of the vulnerability functions. We offer this fourth option because it would be unreasonable for us to prohibit the use of professionally sound structural analysis techniques and impractical for us to specify all professionally sound structural analysis techniques. We urge disclosure of alternative structural analysis techniques to reduce the chance that the consumer of the vulnerability function can overestimate the accuracy of the results.

Collapse is accounted for using the theorem of total probability: the damage factor at a given level of intensity is taken as the collapse probability at that intensity times unity (meaning that we assume a damage factor of 1.0 given collapse) plus the probability of non-collapse times the sum of the story-level damage factors given non-collapse. The collapse probability is quantified by defining a collapse capacity in terms of a lognormal cumulative distribution function of the intensity measure of choice. Default parameter values of both the pushover curve and default collapse capacity are taken from FEMA P-695 (Applied Technology Council 2009) and ASCE 7-10 (American Society of Civil Engineers 2010). If the goal of the analysis is to estimate fatalities, then one estimates fatality rate as the fatality rate conditioned on collapse (recommended by So 2012) times the collapse probability. The analysis ignores fatalities that are not associated with collapse.

Step 4 is to derive a building-level vulnerability function by relating ground motion to story-level motion, calculating story-level loss, and summing over stories. The building-level vulnerability function is factored up to account for the fact that only the top 6 to 8 component categories are inventoried, and may not reflect all the value of the building. The theorem of total probability is then applied to include both the contribution from collapse and the contribution from repair given non-collapse. For fatality risk, one ignores the contribution from non-collapse.

The mean vulnerability function for the asset class is defined as the expected value of repair cost as a fraction of replacement cost new, conditioned on intensity measure level. For fatality risk, the mean vulnerability function for the asset class is defined as the expected value of fatality rate (fraction of occupants killed) conditioned in intensity measure level. If a single index building is used, the mean vulnerability function for the class is taken as the mean vulnerability function for the single, typical index building. If three index buildings are defined, the mean vulnerability function for the class is taken as an equally weighted average of the mean vulnerability functions for the three index buildings. If seven index buildings are used, the mean vulnerability function for the class is taken as a weighted average of the seven using weights derived to match the first several joint moments (mean, variance, skewness, etc.) of the top uncertainties, using a procedure called moment matching.

Uncertainty is quantified in terms of the coefficient of variation (COV) of the damage factor conditioned on the intensity measure level. If one index building is used, the COV is taken using a default function implied by HAZUS-MH (NIBS and FEMA 2012) as inferred in Porter (2010). If three, COV is taken as 1.4 times the coefficient of variation implied by the three index buildings' mean vulnerability functions, that is, 1.4 times the building-to-building variability of mean damage factor. The factor of 1.4 reflects an assumption that within-building uncertainty (from record-to-record variability in ground motion, uncertain damage of building components, and uncertain costs to repair damage) is about equal to the building-to-building variability. If seven index buildings are used, COV is taken as a function of the within-building COVs, the seven mean vulnerability functions, and the moment-matching weight of each index building. An illustrative example is provided. The present method does not attempt to quantify uncertainty in fatality rate.

1.3 Organization of the guidelines

This chapter has introduced the objectives of the methodology and briefly summarized it. Chapter 3 presents a brief review of relevant literature. The details of the procedures are presented in Chapter 4. Chapter 5 presents two illustrative examples. We briefly conclude in Chapter 6. See Chapter 7 for references cited. A series of appendices present supporting information.

2 Literature Review

We reviewed existing literature related to the following topics:

- Dominant nonstructural component categories, that is, categories that tend to dominate nonstructural construction losses and nonstructural construction cost
- Dominant analytical procedures for modelling building component repair cost
- Collapse fragility, which can contribute to loss
- Uncertainty in seismic vulnerability

Dominant nonstructural categories. To begin, we reviewed reconnaissance reports from about 10 recent major earthquakes to identify the component categories that tend to dominate nonstructural losses, either because they are the most numerous kinds of nonstructural components, the most costly, the most fragile, or some combination. These dominant nonstructural components are generally the following:

- Interior partitions
- Exterior closure
- Ceilings
- Heating, ventilation, and air conditioning equipment
- Electrical equipment
- Plumbing equipment

However, we also found that detailed nonstructural elements vary substantially between building types and countries. It is therefore contingent on the analyst to know the asset class well enough to identify the top 5 or so categories of nonstructural component in terms of contribution to construction cost (new). Several sources provide additional guidance. In the United States, RS Means publishes square-foot construction cost manuals that estimate the cost to construct new buildings of several dozen models, where models are generally defined by occupancy, size, exterior wall material, and various options for finishes. Spons offers similar manuals for various countries around the world (e.g., Spons 2013). And local construction contractors possess local expertise that is not published in these manuals.

Dominant analytical procedures for modelling earthquake-induced repair cost. For analytical procedures, we mostly considered FEMA P-58 (Applied Technology Council 2012). While probably too detailed for use here without modification, it represents the state of the art for estimating uncertain future building performance in terms of repair costs, life-safety impacts, and loss of functionality (dollars, deaths, and downtime). For the reader unfamiliar with FEMA P-58, it uses many MDOF nonlinear dynamic structural analyses to estimate uncertain structural response (member forces, deformations, and especially floor-by-floor acceleration and story-by-story peak transient interstory drift) at many intensity levels. Structural analysis is followed by a damage and loss analysis procedure that works at the component level. Building components are defined at a somewhat finer level of detail than NISTIR 6389 (National Institute of Standards and Technology 1999), itself a proposal to subdivide UNIFORMAT II. The FEMA P-58 taxonomy (adapted from Porter 2005) subdivides the NISTIR 6389 categories into subcategories that distinguish among seismic

installation conditions, size, or both. The FEMA P-58 damage-analysis procedure depicts nonstructural damage in discrete damage states using fragility functions derived from experiment, earthquake experience, first principles, and in some cases expert judgment. Repair cost and repair duration are estimated based on construction experience data such as Xactimate or RS Means. FEMA P-58 offers a large though not exhaustive library of damage and repair models. It probably covers a majority of components that would be found in most classes of ordinary residential or commercial US buildings built in the last 50 years. The focus in the FEMA P-58 library is on structural and nonstructural components.

FEMA P-58's fragility database offers fragility functions for many though not all building component categories observed in real buildings around the world. The fragility database is large, too large to duplicate here. It can be obtained from FEMA. Note again however that the FEMA P-58 fragility library generally offers fragility functions for 300 categories of structural components and 450 categories of nonstructural components. Structural components in the FEMA P-58 fragility library include various elements of steel, reinforced concrete, reinforced masonry, and timber construction (though not unreinforced masonry). Nonstructural components in the FEMA P-58 fragility library include various types of exterior closure, interior partitions, ceilings, floor coverings, and a wide variety of mechanical, electrical and plumbing components.

FEMA P-58 builds on a long history of prior art that we will not recap here, other than to refer the interested reader to seminal performance-based engineering work at the Massachusetts Institute of Technology by Czarnecki (1973), application by URS Corporation (Kustu et al. 1982), HAZUS-MH (e.g., Kircher and Whitman 1997), and 2nd-generation performance-based earthquake engineering development for the CUREE-Kajima Joint Research program and the Pacific Earthquake Engineering Research Center (Beck et al. 1999, Porter 2000, Porter et al. 2001, Porter 2003, and Goulet et al. 2007). Nor will we recap work to develop empirical nonstructural vulnerability functions, e.g., Porter et al. (2010). We note that FEMA P-58 offers standard procedures for deriving fragility functions, which are summarized in Porter et al. (2007).

Collapse fragility. Aslani and Miranda (2006) have shown that, at least in the case of an older nonductile reinforced concrete moment-frame building (the Van Nuys California Holiday Inn), collapse can contribute as much to expected annualized repair cost as does damage without collapse. This means a vulnerability function needs to account for collapse fragility. When considering collapse fragility, we reviewed FEMA P-695 (Applied Technology Council 2009), which like many other sources such as Luco et al. (2007) suggests that collapse fragility can be modelled with a lognormal cumulative distribution function using 5% damped elastic spectral acceleration response at the building's approximate fundamental period of vibration. Consistent with US standards (ASCE 2010), these authors explicitly assume the building responds with 100% of its reactive mass participating at the fundamental mode of vibration. FEMA P-695 in particular estimates median collapse capacity \hat{S}_{CT} as the product of:

- C_s Seismic response coefficient. C_s is essentially the design base shear normalized by building weight.
- *R* Response modification factor. *R* is essentially a design ductility demand that approximates ductility capacity. According to ASCE 7-10, *R* varies between 1 and 8, and is tabulated here (in Appendix C) for a variety of materials and structural systems.

- *CMR* Collapse margin ratio. *CMR* is essentially the ratio of collapse capacity to 1.5 times design ground motion, both in terms of $S_a(T_1, 5\%)$. It is typically around 1.5 to 2.0.
- *SSF* Spectral shape factor. *SSF* is a second-order modifier (i.e. second order compared with R and CMR) of roughly 1.15 that accounts for the difference between the shapes of real ground motions, which tend to be peaked, to design spectra.

The same work suggests that the uncertainty in collapse capacity can be modelled as the product of four unit-median lognormal distributions that account for record-to-record variability in ground motion, design requirements, test data, and modelling uncertainties. Without going into detail, the logarithmic standard deviation varies between approximately 0.5 and 0.9. FEMA P-58 (Applied Technology Council 2012) refers to FEMA P-695 as offering the state of the art in collapse fragility. The authors of FEMA P-58 do introduce a judgment-based approach to collapse fragility.

Loss uncertainty. Porter (2010) interprets the procedures in the HAZUS-MH technical manual (NIBS and FEMA 2012) to infer the coefficient of variation of building damage factor as a function of mean damage factor. The relationship derived there is $v(y) = 0.25y^{-0.5}$ where y denotes the mean damage factor (repair cost as a fraction of replacement cost new) and v denotes the coefficient of variation of damage factor. An unpublished analysis of the COV for a class of buildings as implied by Porter (2010) and for individual, particular buildings as implied by Porter et al. (2002) suggests that the building-to-building variability of damage factor approximately equals the uncertainty in damage factor for an individual building. These facts will be useful when estimating the coefficient of variation of damage factor if too few index buildings are used to explicitly or completely estimate uncertainty.

Fatality rate. As part of the efforts of the GEM Vulnerability Consortium, So (2012) provides an extensive literature review of earthquake-induced casualties, along with her own synthesis of how to estimate fatalities in collapsed buildings. The interested reader is referred to that work for detail.

3 Analytical Procedure

3.1 Overview of uncertainty propagation used here

This section provides an overview of the procedure used here to account for uncertainty in the seismic vulnerability function of a class of buildings. There are many ways to propagate uncertainty in seismic vulnerability functions for classes of buildings: Monte Carlo simulation, Latin Hypercube simulation, moment matching, and probably others. One could try to propagate all uncertainties at every stage in each sample building. These guidelines propose procedures with low, medium, or high level of effort.

The low-effort procedure requires no uncertainty propagation by the analyst: a single index building that the analyst considers to be typical of the class is modelled. Its mean damage factor is calculated, and this is taken as the mean damage factor of the class. A coefficient of variation of damage factor for the class is assumed, taken from a prior study of uncertainty in HAZUS-MH building classes (Porter 2010). See Figure 1a.

The medium-effort procedure requires the analyst to model poor, typical, and superior-quality index buildings that represent the analyst's opinion of the range of performance of the class. The simple average of their mean damage factors is taken as the mean damage factor for the class. The sample coefficient of variation of the three index buildings is taken as the between-building variability of the vulnerability of the class. The analyst assumes that the within-building variability approximately equals the between-building variability (an assumption supported by limited testing), and that the variance of the two can be summed to calculate the variance of the vulnerability function for the class, which equates with multiplying the sample COV by 1.41, as shown in Figure 1b.

The high-effort procedure is called moment matching; see Porter and Cho (2013) in Appendix A for details. It is a generalization of Gaussian quadrature that is commonly used in finite element analysis. Briefly, the analyst selects the three leading factors or variables that influence the vulnerability of the class. The analyst calculates their joint probability distribution from a large sample of buildings in the class, and then selects seven sets of all three variables. Each sample has a fixed value of each of the three factors, and an associated weight for the sample. The samples and weights are selected so that the first five moments (mean, variance, skewness, etc.) of the weighted sample set match the first five moments of the population as a whole. When the weighted average of the vulnerability functions of the seven index buildings is calculated, it will estimate the mean damage factor of the class with 5th order accuracy, that is, like a Taylor Series expansion to the 5th term. Between-building variability) is accurate to 2nd order. Within-building variability (the uncertainty in the vulnerability of a given index building) is calculated through Monte Carlo simulation of the ground-motion time history, damage, and loss. The mean damage factor and coefficient of variation of damage factor is calculated for each of the seven index buildings. The weighted combination of the probability density functions of the seven index buildings is integrated to calculate the coefficient of variation of the probability density functions of the seven index buildings is integrated to calculate the coefficient of variation of the probability density functions of the seven index buildings is integrated to calculate the coefficient of variation of the

vulnerability function of the class, considering both within-building and between-building variability. See Figure 1(c) for illustration and Appendix A for theoretical justification. In Figure 1(c), the dashed line represents the coefficient of variation of damage factor for the building class. The heavy line labelled "Asset class MDF" represents the weighted average of the remaining, lighter lines. The lighter lines represent the mean vulnerability function for each of the seven index buildings. In this particular case, the seven index buildings include a building with relatively low design base shear, one with high design base shear, one with a low degree of vertical irregularity, one with a high degree, one that represents a shorter member of the class, one that represent a taller member of the class, and one base case that has neither low nor high design base shear, neither low nor high vertical irregularity, and a height near the median of the class.

Note that moment matching does not require that the joint distribution of the variables be continuous, unimodal, or that the variables be independent. The present work is limited to variables that are independent, but Ching et al. (2008) show how to rotate axes to account for dependence between the variables.

A reasonable alternative to moment matching is to use a procedure called class partitioning. See Diday et al. (2005) for detail on this approach. Class partitioning is briefly summarized in Appendix D, but is not illustrated here.



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3.2 Step 1: Define the asset class at risk with one, three, or seven index buildings

This procedure consists of four steps, beginning with definition of the asset class via a few attributes of either one, three, or seven index buildings.

How many index buildings? If the user is constrained regarding time and does not mind treating uncertainty with a default relationship, use a single index building that represents a median or typical case, as described in more detail shortly. With more time, one can explicitly propagate uncertainty. The user can select the characteristics and nonstructural inventory for three index buildings: one a poor case, with relatively fragile components, a typical case like the one just mentioned, and a superior case, with relatively rugged or

seismically restrained components. Some judgment is required to establish the characteristics of these variants. Finally, with seven index buildings and some additional Monte Carlo simulation, one can explicitly propagate uncertainty associated within and between specimens. Again, details are presented shortly.

One index building. Select the taxonomy attributes of the asset class of interest (E.g., lowrise, midrise, or highrise building? Reinforced concrete, steel, or masonry structural material? Residential, commercial, or industrial occupancy? And so on. The user must decide which attributes to use to define the asset class; we provide no guidance on that point.) Create an inventory of building components for a single index building, that is, for a typical case, with a mix of components that are neither much more rugged than typical nor very fragile. By "component" we mean a category of parts of a building as defined by the FEMA P-58 taxonomy (an extension of NISTIR 6389, itself an extension of UNIFORMAT II). Each line of the fragility tab in the spreadsheet represents a component category.

The analyst will see in the FEMA P-58 spreadsheet that most component categories have multiple variants, some more fragile than others, some less. The median capacity of the first damage state is an indicator of the relative fragility of a component. By "inventory" is meant the quantities of the components of the building on a story-by-story basis. Use the FEMA P-58 taxonomy or any other convenient taxonomy where the replacement cost and fragility of each class of component can be estimated. For present purposes, these guidelines adopt the FEMA P-58 component taxonomy by reference. The analyst should seek the latest version of the fragility database from the Federal Emergency Management Agency. To create an inventory, document the design of the index building as shown in Table 1 (overall description of the index building), Table 2 (identify top 6 components), Table 4a-f (fragility functions and unit repair costs for top 6 components), and Table 5.

Three index buildings. With more time or need to explicitly propagate uncertainty, the analyst can select the characteristics and inventory for three index buildings: one a poor case, with relatively low design base shear and fragile components, a typical case like the one just mentioned, and a superior case, with relatively rugged or seismically restrained components and relatively high design base shear. Other variables to consider to differentiate poor, typical and superior quality index buildings include number of stories, presence of vertical irregularity, and presence of plan irregularity. The poor, typical, and superior-quality index buildings should represent in the analyst's mind cases where repair cost would be exceeded respectively by 10%, 50%, and 90% of buildings of the same classification according to the GEM building taxonomy. (We chose these three percentiles because of their value in quadrature. That is, these percentiles are chosen so that when the vulnerability functions for the three index buildings are equally weighted, the mean and variance of the weighted average approximate those of a continuous lognormal distribution whose coefficient of variation is near 0.5, a reasonable typical value per Porter [2010].) This approach will quantify variability of vulnerability between specimens in the asset class. The uncertainty in vulnerability within an individual specimen will be assumed to be equal to the between-specimen variability. Document the design of each index building with one set of tables, Table 1 (overall description of the index building), Table 2 (identify top 6 nonstructural components), Table 3 (identify top 2 structural components), Table 4a-h (fragility functions and unit repair costs for top 6 nonstructural components and top 2 structural components), and Table 5 (component inventory by story). That is, complete Table 1 through Table 5 for each index building.

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Seven index buildings. Finally, the analyst can explicitly quantify within-specimen variability using Monte Carlo simulation and more thoroughly quantify the between-specimen variability using a procedure called moment matching that we specify here. (A reasonable alternative approach to quantifying betweenspecimen variability is to use a procedure called class partitioning, but we do not specify that procedure here.) This approach requires advanced skills in simulation and some skills to implement moment matching. The necessary probability information is provided here, but not details of the Monte Carlo simulation. If the analyst does not already possess the required skills, this approach is not recommended. This approach requires the analyst to quantify the probability distribution within the asset class of three key attributes: number of stories, degree of vertical irregularity, and design base shear. (The analyst is free to choose other features that he or she believes more strongly influence variability between specimens in an asset class, such as degree of plan irregularity, but the analyst will need to quantify the probability distribution of each such feature.) All these probability distributions must be estimated or compiled by observation of many samples. The procedures for making the necessary observations and selecting the values of the key attributes are detailed in the manuscript in Appendix A. The tables for defining the samples of the asset class are provided in Appendix B, along with notes defining number of stories, degree of plan irregularity, and degree of vertical irregularity, as well as notes about probability distributions for structural analysis, damage analysis, and loss analysis. The reader may wonder why not use Monte Carlo Simulation throughout, including in the design of index buildings? The reason is pragmatic: to create a structural model is time consuming compared with the rest of the analysis procedure proposed here. Moment matching allows the analyst to create a small number of structural models that sample over a few features that matter most to structural response, far more efficiently than does Monte Carlo simulation. See Ching et al. (2008) and Porter and Cho (2013) for details. Again, class partitioning, as summarized in Appendix D and detailed in Diday et al. (2005) is a reasonable alternative.

Note that for each index building, the analyst will also define three variants: one with poor quality components, one with a typical mix of poor and superior quality components, and one with superior quality components. By poor and superior quality components is meant that some categories of components can be installed with greater or less seismic resistance. For example, furniture can be either secured to the building frame (which would be superior quality) or freestanding (which would be poor quality). Equipment can be anchored or not anchored. Wallboard partition can be partial height (which tends to tolerate greater drift without damage) or full height (which tends to tolerate less drift without damage). The analyst must judge for the building class what mix of components would represent poor, typical, and superior quality. The choice will usually depend on local construction practices and how common it is for building owners in the region to retrofit buildings of that class to make them more seismically resistant. Note also that, as used here, a variant is not the same as an index building. We are not talking about 21 index building only differ in their mix of fragility functions and associated repair costs.

Asset class (e.g., material, LLRS, height category, occupancy)	
Structural material (if used, from GEM building	
taxonomy)	
Lateral load resisting system (if used, from GEM	
building taxonomy)	
Broad category (choose one)	Frame, shearwall, mixed
Height category (if used, from GEM building	
taxonomy)	
Occupancy (if used, from GEM building taxonomy)	
Other attribute 1	
Other attribute 2	
Other attribute 3	
Index building quality (if using 3 index buildings; choose one)	Poor, typical, or superior
Index building name (if a particular real building is selected)	
Index building model (if using local per-square-meter cost	
manual)	
Stories (number of stories above grade)	
Story height (m) (e.g., building height divided by number of	
stories)	
Building height (m)	
Design year (year when the design as assumed to be	
completed)	
Construction year (if the building is a real one)	
Labor cost as a fraction of total labor + material in	
construction cost	
Local labor cost as a fraction of US labor cost	
Estimated fundamental period of vibration T, sec	
Gamma (default = 1.3)	
Median collapse capacity, S_{CT} , g, IM = Sa(1, 5%), geomean	
Logarithmic standard deviation of collapse capacity (default	
0.8)	
Design base shear as fraction of building weight Cs	
Cost manual reference (if used)	
Total building cost (currency per m)	
Total building floor area (m)	
Fraction f inventory construction cost (currency)	
Fraction J_1 inventory construction cost as fraction of	
construction cost represented by the components considered	
horo	
licie.	

Table 1. Index building definition (1 or 3 index buildings)

Table 2. Ranking of nonstructural components in decreasing order of contribution to construction cost (1 or 3 index

buildings)

Rank ¹	1	2	3	4	5	6
Description ²						
NISTIR 6389 class ID ³						
FEMA P-58 class ID ⁴						
Demand param (PFA or PTD) ⁵						

1 Component categories in decreasing order of construction cost

2 A brief text description of the component in question

3 The alphanumeric code for the component type per NISTIR 6389 (National Institute of Standards and Technology 1999)

4 The alphanumeric code for the component type per FEMA P-58 (Applied Technology Council 2012)

5 Demand parameter, typically either peak floor acceleration or peak transient interstory drift at the level of the component

Rank	1	2
Description		
NISTIR 6389 class ID		
FEMA P-58 class ID		
Demand param (PFA or PTD)		

Table 3. Ranking of structural components in decreasing order of contribution to construction cost (1 or 3 index buildings)

See notes for Table 2.

Table 4a. Fragility functions and unit repair costs, nonstructural component rank 1 (1 or 3 index buildings)

	Nons	tructural Component S	Specification, Rank #1	
NISTIR Class ¹		Description ¹		
FEMA P-58		Unit ²		
Class ¹				
Demand	PFA or PTD	Reference ³		
param ¹				
	Fragility function	n	Repair co	st by damage state
Damage state	Fragility function Median capacity ⁴	n Log std deviation⁵	Repair co P ₅₀ (median cost) ⁶	st by damage state Log std deviation ⁷
Damage state	Fragility function Median capacity ⁴	n Log std deviation ⁵	Repair co P ₅₀ (median cost) ⁶	st by damage state Log std deviation ⁷
Damage state 1 2	Fragility functio Median capacity ⁴	n Log std deviation ⁵	Repair co P ₅₀ (median cost) ⁶	st by damage state Log std deviation ⁷
Damage state 1 2 3	Fragility functio Median capacity ⁴	n Log std deviation ⁵	Repair co P ₅₀ (median cost) ⁶	st by damage state Log std deviation ⁷

1 See notes for Table 2

2 The units by which the component is measured, e.g., per each, or per 100m of wall, etc.

3 Reference to fragility and repair-cost data. Default is PACT 1.0 (Applied Technology Council 2012)

4 Value of demand parameter associated with 50% probability of reaching or exceeding the specified damage state

5 Logarithmic standard deviation of capacity, i.e., the standard deviation of the natural logarithm of capacity

6 50th percentile of uncertain cost to repair one unit of the component from the specified damage state

7 Logarithmic standard deviation of repair cost, i.e., the standard deviation of the natural logarithm of the cost to repair one unit of the component from the specified damage state

Nonstructural Component Specification, Rank #2							
NISTIR Class		Description					
FEMA P-58		Unit					
Class							
Demand	PFA or PTD	Ref (default Pact					
param		1.0)					
	Fragility function	n	Repair co	st by damage state			
Damage state	Median capacity	Log std deviation	P ₅₀ (median cost)	Log std deviation			
1							
2							
3							
4							

Table 4b. Fragility functions and unit repair costs, nonstructural component rank 2 (1 or 3 index buildings)

For further detail, see notes for Table 4a

Table 4c. Fragility functions and unit repair costs, nonstructural component rank 3 (1 or 3 index buildings)

Nonstructural Component Specification, Rank #3									
NISTIR Class		Description							
FEMA P-58		Unit							
Class									
Demand	PFA or PTD	Ref (default Pact							
param		1.0)							
	Fragility functio	n	Repair co	st by damage state					
Damage state	Modian canacity	Las shalalay dation							
0	ineutan capacity	Log std deviation	P ₅₀ (median cost)	Log std deviation					
1		Log std deviation	P ₅₀ (median cost)	Log std deviation					
1 2		Log std deviation	P ₅₀ (median cost)	Log std deviation					
1 2 3			P ₅₀ (median cost)	Log std deviation					

For further detail, see notes for Table 4a

Table 4d. Fragility functions and unit repair costs, nonstructural component rank 4 (1 or 3 index buildings)

Nonstructural Component Specification, Rank #4							
NISTIR Class		Description					
FEMA P-58		Unit					
Class							
Demand	PFA or PTD	Ref (default Pact					
param		1.0)					
Fragility function Repair cost by damage state							
	Fragility function	n	Repair co	st by damage state			
Damage state	Fragility function Median capacity	n Log std deviation	Repair co P ₅₀ (median cost)	st by damage state Log std deviation			
Damage state	Fragility function Median capacity	n Log std deviation	Repair co P ₅₀ (median cost)	st by damage state Log std deviation			
Damage state 1 2	Fragility function Median capacity	n Log std deviation	Repair co P ₅₀ (median cost)	st by damage state Log std deviation			
Damage state 1 2 3	Fragility functio Median capacity	n Log std deviation	Repair co P ₅₀ (median cost)	st by damage state Log std deviation			

For further detail, see notes for Table 4a

Nonstructural Component Specification, Rank #5							
NISTIR Class	Description						
FEMA P-58		Unit					
Class							
Demand	PFA or PTD	Ref (default Pact					
param		1.0)					
	Fragility function	n	Repair co	st by damage state			
Damage state	Median capacity	Log std deviation	P ₅₀ (median cost)	Log std deviation			
1							
2							
3							
4							

Table 4e. Fragility functions and unit repair costs, nonstructural component rank 5 (1 or 3 index buildings)

For further detail, see notes for Table 4a

Table 4f. Fragility functions and unit repair costs, nonstructural component rank 6 (1 or 3 index buildings)

Nonstructural Component Specification, Rank #6									
NISTIR Class		Description							
FEMA P-58		Unit							
Class									
Demand	PFA or PTD	Ref (default Pact							
param		1.0)							
	Fragility functio	n	Repair co	st by damage state					
Damage state	Fragility functio Median capacity	n Log std deviation	Repair co P ₅₀ (median cost)	st by damage state Log std deviation					
Damage state	Fragility functio Median capacity	n Log std deviation	Repair co P ₅₀ (median cost)	st by damage state Log std deviation					
Damage state 1 2	Fragility functio Median capacity	n Log std deviation	Repair co P ₅₀ (median cost)	st by damage state Log std deviation					
Damage state 1 2 3	Fragility functio Median capacity	n Log std deviation	Repair co P ₅₀ (median cost)	st by damage state Log std deviation					

For further detail, see notes for Table 4a

Table 4g. Fragility functions and unit repair costs, structural component rank 1 (1 or 3 index buildings)

Structural Component Specification, Rank #1							
NISTIR Class		Description					
FEMA P-58		Unit					
Class							
Demand	PFA or PTD	Ref (default Pact					
param		1.0)					
Fragility function Repair cost by damage state							
	Fragility function	n	Repair co	st by damage state			
Damage state	Fragility function Median capacity	n Log std deviation	Repair co P ₅₀ (median cost)	st by damage state Log std deviation			
Damage state	Fragility function Median capacity	n Log std deviation	Repair co P ₅₀ (median cost)	st by damage state Log std deviation			
Damage state 1 2	Fragility function Median capacity	n Log std deviation	Repair co P ₅₀ (median cost)	st by damage state Log std deviation			
Damage state 1 2 3	Fragility function Median capacity	n Log std deviation	Repair co P ₅₀ (median cost)	st by damage state Log std deviation			

For further detail, see notes for Table 4a

Structural Component Specification, Rank #2							
NISTIR Class		Description					
FEMA P-58		Unit					
Class							
Demand	PFA or PTD	Ref (default Pact					
param		1.0)					
Fragility function Repair cost by damage state							
Damage state	Median capacity	Log std deviation	P ₅₀ (median cost)	Log std deviation			
1							
2							
3							
4							

Table 4h. Fragility functions and unit repair costs, structural component rank 2 (1 or 3 index buildings)

For further detail, see notes for Table 4a

	Nonstructural components						Structural of	omponents
Rank:	1	2	3	4	5	6	1	2
Name								
Unit								
Story			Quantit	ty (total)				
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								

Table 5. Component inventory by story (1 or 3 index buildings)

Add rows as necessary

3.3 Step 2: Derive story-level vulnerability functions

This step is described in detail only for the cases of 1 and 3 index buildings. It is assumed that only advanced analysts will examine 7 index buildings using Monte Carlo simulation (MCS), and that these advanced users can infer the necessary MCS tasks from the following. In this step, one calculates the story-level vulnerability function for all acceleration-sensitive components on each story, and all drift-sensitive components on each story. The vulnerability function is evaluated at several levels of floor acceleration. Let

 i_a = index to acceleration-sensitive component types, $i \in \{1, 2, ..., N_a\}$

 i_r = index to drift-sensitive component types, $i \in \{1, 2, ..., N_r\}$

 N_a = number of acceleration-sensitive component types included in the inventory

 N_r = number of drift-sensitive component types included in the inventory

 s_{ha} = peak floor acceleration at floor h, to be evaluated at $s_{ha} \in \{0, 0.01, 0.02, \dots 3.00\}$

 s_{hr} = peak transient drift at story h, to be evaluated at $s_{hr} \in \{0, 0.005, 0.010, \dots 0.20\}$ (the drifts reach 0.20 because woodframe buildings can tolerate drifts in excess of 10% without collapse)

 $Ca_{h,ia}(s) =$ (uncertain) repair cost of acceleration-sensitive components of type i_a at floor h

 $Cr_{h,ir}(s)$ = (uncertain) repair cost of acceleration-sensitive components of type i_r at floor h

 $E[Ca_{h,ia}(s)]$ = expected value of repair cost of acceleration-sensitive components of type *i* at floor *h*

 $E[Cr_{h,ir}(s)] =$ expected value of repair cost of drift-sensitive components of type *i* at floor *h*

Evaluating $E[Ca_{h,ia}(s)]$ and $E[Cr_{h,ir}(s)]$ involves the fragility functions and unit repair costs for each fragility function. Here, it is assumed that all fragility functions are in the form of lognormal cumulative distribution functions. It is also assumed that all components with multiple damage states in addition to the undamaged state have sequential fragility functions, that is, a component must pass through damage state *d* before it can reach damage state *d*+1. There are also cases of components with either simultaneous or mutually exclusive and collectively exhaustive (MECE) damage states. For treating these, see Box 1. Note that there is frequent use here of mean, median, standard deviation, logarithmic standard deviation, and coefficient of variation. See Box 2 for conversions among these parameters.

Box 1: components with simultaneous or MECE damage states

There are cases of components with either simultaneous or mutually exclusive and collectively exhaustive (MECE) damage states. In the FEMA P-58 fragility library, elevators are an example of the first case, steel beams are an example of the second. These have a single fragility function for the occurrence of any damage, and a probability mass function (PMF) for the alternative damage states. In the case of simultaneous, the probabilities in the PMF sum to a value greater than 1.0. In the case of MECE damage states, they sum to 1.0. To deal with either case, we simplify the loss analysis by assigning an expected value of unit repair cost (cost to restore a unit of the component type to the undamaged state) and the coefficient of variation of unit repair cost as shown in Equation (1) and (2), respectively. Again, these two equations are only needed in the case of simultaneous or MECE damage states. In these equations, $m_{i,1}$ and $v_{i,1}$ respectively denote the mean and coefficient of variation of repair cost for a unit of component of category i given that any damage occurs; d denotes the possible damage states, N_D denotes the number of possible damage states; $p_{i,d}$ denotes the probability that a component of category i, once damaged, is in damage state d; $\dot{m}_{i,d}$ denoted the expected value of repair cost given that a component of category i, once damaged, is in damage state d; and $\sigma_{i,d}$ denotes the standard deviation of repair cost given that a component of category i, once damaged, is in damage state d. Henceforth the component with simultaneous or MECE damage states can be treated like one with a single possible damage state, i.e., $N_D = 1$.

n

$$n_{i,1} = \sum_{d=1}^{N_D} p_{i,d} \cdot \dot{m}_{i,d}$$
(1)

$$y_{i,1} = \frac{1}{m_{i,1}} \sqrt{\sum_{d=1}^{N_D} p_{i,d} \cdot \left(\sigma_{i,d}^2 + \left(\dot{m}_{i,d} - m_{i,1}\right)^2\right)}$$
(2)

There are a few components in the FEMA P-58 database that have a more-complicated set of damage states, e.g., several sequential damage states in which one is a MECE combination of two or more finer damage states. These combinations can be collapsed so that their repair cost can be represented as if the combination were a single equivalent sequential damage state, using Equations (1) and (2). See Box 3 for additional guidance on estimating local repair costs.

In the case of lognormal fragility functions and sequential damage states (or simultaneous or MECE treated as described in Box 1), the mean fraction of specimens of type *i* that are damaged in damage state *d* is the expected per-specimen failure probability (denoted here by $p_{i,d}$) and given by Equation (3).

$$p_{i,d}(s) = \Phi\left(\frac{\ln\left(s/\theta'_{i,d}\right)}{\beta'_{i,d}}\right) - \Phi\left(\frac{\ln\left(s/\theta'_{i,d+1}\right)}{\beta'_{i,d+1}}\right) \qquad d < N_D$$
$$= \Phi\left(\frac{\ln\left(s/\theta'_{i,d}\right)}{\beta'_{i,d}}\right) \qquad d = N_D \qquad (3)$$
$$\ge 0$$

where

$$\beta'_{i,d} = \sqrt{\beta^2_{i,d} + \beta^2_m} \tag{4}$$

$$\theta_{i,d}' = \theta_{i,d} \cdot \exp\left(0.84 \cdot \left(\beta_{i,d}' - \beta_{i,d}\right)\right)$$
(5)

In Equation (3), *s* denotes peak floor acceleration in the case of acceleration-sensitive components and peak transient drift in the case of drift-sensitive components; $\vartheta_{i,d}$ and $\beta_{i,d}$ are median and logarithmic standard deviation of capacity of the fragility function for component category *i*, damage state *d*, taken from Table 4a through Table 4f, and N_D is the number of damage states in addition to the undamaged state. The floor of zero in Equation (3) is included to allow for different values of β in the denominators, which can lead to fragility functions that cross. The parameter β_m in Equation (4) adds uncertainty associated with approximations in the structural model. In the case of a pushover structural analysis, β_m accounts for the approximation of using a pushover rather than multiple nonlinear dynamic analyses ($\beta_m = 0.3$, from FEMA P-695 [Applied Technology Council 2009]). If the analyst employs nonlinear dynamic structural analysis, take $\beta_m = 0$ as in FEMA P-58. Supporting documentation regarding the errors inherent in nonlinear static pushover based methods of analysis can be found in NIST GCR 10-917-9 (National Institute of Standards and Technology, 2010).

$$\beta_m = 0.4$$
 simplified
= 0.3 nonlinear static (6)
= 0 nonlinear dynamic

Because increasing uncertainty with $\beta_m > 0$ will tend to bias risk upwards if one does not also adjust the median capacity, $\theta_{i,d}$ is adjusted to ensure that the fragility function with the higher β is rotated about the 20th percentile rather than the 50th, that is, the excitation associated with 20% failure probability is unchanged, hence the modification in Equation (5). That is, we increase the median from $\theta_{i,d}$ to $\theta_{i,d}'$ per
Equation (5) to acknowledge greater uncertainty in fragility when using a more-approximate structural model, without also biasing the loss estimate. (We examined a range of rotation points between the 10th and 50th percentiles, and found that, for a range of realistic hazard and fragility functions, rotating about the 20th percentile introduces the least bias.)

The mean vulnerability function per component of category *i* is given by

$$E\left[C_{i}\left|S_{h,i}=s\right]=N_{i}\cdot\sum_{d=1}^{N_{D}}p_{i,d}\left(s\right)\cdot m_{i,d}$$
(7)

where $m_{i,d}$ denotes the mean repair cost per unit of component category *i*, damage state *d*. It can be calculated from the median unit repair cost P_{50} and logarithmic standard deviation of unit repair cost *b*, as follows.

$$m_{i,d} = P_{50,i,d} \exp(0.5 \cdot b^2)$$
 (8)

See Box 2 for additional guidance on estimating $m_{i,d}$. Now estimate the story-level vulnerability for acceleration sensitive components as

$$E[C_{a}|S_{h,a} = S_{a}] = \sum_{i=1}^{N_{a}} E[C_{i}|S_{h,a} = S_{a}]$$
(9)

Where C_a denotes the repair cost for acceleration-sensitive components. Repeat for drift-sensitive components:

$$E\left[C_{r}|S_{h,r}=s_{r}\right] = \sum_{i=1}^{N_{r}} E\left[C_{i}|S_{h,r}=s_{r}\right]$$
(10)

Where

 C_a = repair cost of acceleration-sensitive components on the given story

 C_r = repair cost of drift-sensitive components on the given story

E[X|Y] = expected (mean) value of X given conditions Y

 $S_{h,a}$ = uncertain story-level acceleration at floor h

$$s_a$$
 = a particular value of $S_{h,a}$

 $S_{h,r}$ = uncertain story-level drift ratio (unitless) at story h

$$s_r$$
 = a particular value of $S_{h,r}$

i = an index to the component categories present on the story

 N_a = number of acceleration-sensitive component categories present on the story

 N_r = number of drift-sensitive component categories present on the story

 $N_{i,h}$ = quantity of components of category i on story h, from Table 5.

 Φ = standard normal cumulative distribution function evaluated at the term in parentheses

 ϑ_i = median capacity of the component category, from Table 4a-f.

 θ_i = logarithmic standard deviation of the capacity of the component category from Table 4a-f.

Evaluate the mean story-level vulnerability function at floor accelerations $s \in \{0.01, 0.02, ..., 3.0g\}$ and drifts $s \in \{0.001, 0.02, ..., 0.10\}$ for each story h of each index building. These calculations can be done in a spreadsheet, without structural analysis.

Box 2: mean, median, standard deviation, logarithmic standard deviation, and coefficient of variation

Here are some useful conversions among the parameters of lognormally distributed random variables. Let

m = mean (same as "expected value")

 ϑ = median (same as "50th percentile")

- σ = standard deviation
- β = logarithmic standard deviation
- v = coefficient of variation

$$m = \theta \cdot \exp(0.5 \cdot \beta^2) \tag{11}$$

$$\theta = \frac{m}{\sqrt{1 + v^2}} \tag{12}$$

$$\sigma = m \cdot \sqrt{\exp(\beta^2) - 1} \tag{13}$$

$$v = \sqrt{\exp(\beta^2) - 1} \tag{14}$$

$$\beta = \sqrt{\ln\left(1 + v^2\right)} \tag{15}$$

Box 3: local repair costs

Note the requirement for unit repair costs $m_{i,d}$ expressed in the currency and local costs of the country of interest. The analyst can seek this information from local construction contractors, using the description of the repair efforts in associated with the damage state, such as from the FEMA P-58 PACT database (Applied Technology Council 2012). In a few cases, resources such as Xactimate (Xactware 2012) are available to provide local cost information. In case the analyst has no access to resource other than FEMA P-58 PACT database, we offer the following very approximate approach to estimating unit repair costs. Let

r_{lab} = ratio of hourly cost for local construction labor to the hourly cost for US construction labor. Estimate the hourly cost of US construction cost labor by referring to the US Bureau of Labor Statistics' cost data for construction laborers at http://www.bls.gov/oes/current/oes472061.htm. Other countries may have similar national data to estimate the hourly cost of local construction labor. If not, refer to a local construction contractor.

 f_{lab} = fraction of US unit repair cost associated with labor. Absent better information, we recommend these default values:

= 0.9 efforts requiring repairing but not replacing architectural and structural components, mechanical, electrical, or plumbing equipment

- = 0.5 efforts requiring replacing architectural and structural components
- = 0.1 efforts requiring replacing mechanical, electrical, and plumbing equipment

 $m_{i,d(US)}$ = mean unit repair cost in the US, e.g., according to the FEMA P-58 PACT database.

 $m_{i,d}$ = mean unit repair cost in the local context

$$m_{i|d} = m_{i,d(US)} \left(1 - f_{lab} \right) + m_{i,d(US)} \left(f_{lab} \cdot r_{lab} \right)$$
(16)

 $P_{90(US)} = 90^{\text{th}}$ percentile of unit repair cost in the US, from FEMA P-58 PACT database

 $P_{10(US)} = 10^{\text{th}}$ percentile of unit repair cost in the US, from FEMA P-58 PACT database

 $\sigma_{i,d(US)}$ = standard deviation of unit repair cost in the US

$$\sigma_{i,d(US)} = \frac{1}{2.56} (P_{90} - P_{10})$$
⁽¹⁷⁾

 $\sigma_{i,d}$ = standard deviation of unit repair cost in the local context

$$\sigma_{i,d} = \sigma_{i,d(US)} \cdot \frac{m_{i,d}}{m_{i,d(US)}}$$
(18)

3.4 Step 3: Structural analysis

3.4.1 General requirements for structural analysis

The analytical estimation of seismic losses is based on combining probabilistic seismic hazard analysis with seismic vulnerability functions. This document focuses on the latter, but it is important to ensure that these two complex calculations mesh well. The point of contact between them is an interface variable that links seismic hazard with structural response. The variable is referred to here as the intensity measure (IM), referring to a measure of the intensity of ground motion. A rather unobvious requirement for the IM is that it needs to crystallize all the seismological properties of ground motion to make any assessment (a) practical (b) efficient and (c) sufficient with respect to the underlying issues related to site and source (Luco and Cornell 2007). Practicality necessitates the use of IMs for which ground motion prediction equations (GMPEs, also known as attenuation relationships) are available. This currently restricts the analyst to a choice among peak ground acceleration, peak ground velocity, peak ground displacement. and spectral acceleration response. The latter is where most research is currently focused among engineering seismologists. A more efficient IM is one that produces a lower uncertainty in structural response conditioned on seismic source and path parameter values in a fixed and ideally small number of time-history structural analyses. A sufficient IM is one that accounts for all source and path parameters that strongly affect structural response, so that conditioned on IM, response is independent of other source and path parameters. An IM that leaves the structural response sensitive to other parameters can cause unwanted bias to creep into loss estimates wherever the ground motion characteristics do not match the source and site requirements for the building and IM level that is being considered.

Together with the selection of IM comes the selection and scaling of ground motions for use in time-history structural analysis. Since vulnerability functions are needed for a large range of IM values, a structural model needs to be subjected to a wide range of IM values that will force it to show its full range of response and loss, from elastic response to global collapse. Because of limitations in the catalogue of ground motion recordings, it is often desirable to be able to modify (i.e. scale) a record to display the desired IM level. (Using artificial accelerograms is another potential approach, but it is not recommended for general use at this point unless the analyst possesses the necessary expertise). A sufficient IM theoretically allows unrestricted scaling of ground motions to match any IM level. In reality, though, no single IM is perfect. Therefore, exercising at least a minimum of care in selecting ground motions is advised. Since vulnerability functions are usually developed to be applicable to wide geographic regions, it is often not possible to select ground motion time histories according to the most recent research findings, using e.g. the conditional mean spectrum (Baker and Cornell, 2008), or incorporating near-source directivity. In general, our recommendation is to use the

best possible IM that will allow a wide range of scaling, plus a suite of relatively strong ground-motion records recorded on firm soil. Whenever sufficient information exists about the dominant seismic mechanism or site soil in the region for which the vulnerability curve is developed (e.g. crustal earthquakes in western USA or soft soil in Mexico City), it may help an experienced analyst in choosing at least some of the characteristics of the records to use.

Being aware of the requirements, let us now consider the choice of IM. The most popular option is $S_a(T_1, 5\%)$, i.e. the 5% damped elastic spectral acceleration response at the period of interest (usually a structure's estimated fundamental period of vibration, T_1). $S_a(T_1, 5\%)$ has been found to be both efficient and sufficient for first-mode-dominated, moderate period structures when directivity is not present, but has often been criticized for lack in sufficiency wherever large scale factors (higher than, say 3.0) are employed. This is mainly the case for modern structures that need considerably intense ground motions to experience collapse. On the other hand, this is rarely the case for older and deficient buildings. A good alternative for single buildings is $S_{agm}(T_i)$, defined here as the geometric mean of the 5% damped elastic spectral acceleration response at the estimated fundamental periods of the index buildings i. Use of this IM significantly improves the efficiency and the sufficiency of the estimation. It also remains practical as a GMPE for $S_{aqm}(T_i)$ is easily estimated from existing GMPEs. It offers a considerable extension to the applicability of scaling (Vamvatsikos and Cornell, 2005a; Bianchini et al. 2009), and is the recommended approach whenever undertaking nonlinear dynamic structural analysis (e.g., IDA). Simpler, nonlinear static procedure methods are based on $S_{a}(T_{1}, 5\%)$ by default as a single-degree-of-freedom approximation lies at their basis. This cannot be changed and it is an often disregarded constraint of the pushover analysis. Its implications are simply accumulated together with the other (and often larger) errors that the approximate nature of the pushover incurs. It is doubtful whether the benefits of $S_{aam}(T_i)$ can be realized here, thus $S_a(T)$ at a common period T will be the recommended choice for parameterizing the vulnerability curves from such methods.

We offer a simplified default structural analysis procedure in Section 3.4.2. We offer guidance for nonlinear pseudostatic (pushover) structural analysis in Section 3.4.3 and for incremental dynamic analysis in Section 3.4.4. Or the analyst can use any other familiar structural analytical procedure options for performing the structural analysis. The objective of the structural analysis is to estimate geometric mean peak floor acceleration at each floor of the building, peak transient drift ratio at each story of the building, and probability of collapse, all three measures at each of many intensity measure levels.

- x = intensity measure level. Select the intensity measure is most of interest. The default used here is as follows:
 - = $S_a(0.3 \text{ sec}, 5\%)$ for index buildings with $T_1 \le 0.5 \text{ sec}$
 - = $S_a(1.0 \text{ sec}, 5\%)$ for index buildings with 0.5 sec < $T_1 \le 2.0 \text{ sec}$
 - = $S_a(3.0 \text{ sec}, 5\%)$ for index buildings with 2.0 sec < T_1
 - = $S_a(T_{1m}, 5\%)$ for index buildings analysed using nonlinear dynamic or static analysis with scaling factors < 3.0,

= S_{agm} for index buildings analysed using nonlinear dynamic analysis regardless of scaling level

$$S_{agm} = \left[\prod_{i=1}^{n} S_a(T_i, 5\%)\right]^{1/n}$$
, i.e., the geometric mean of the 5% damped spectral acceleration response at *n*

periods of vibration, T_i (*i* = 1,...*n*)

Suggested T_i values for highrise buildings are the following five: { $T_{2m\nu} min(1.5 \cdot T_{2m\nu} 0.5 \cdot (T_{2m} + T_{1m}))$, $T_{1m\nu} 1.5 \cdot T_{1m\nu}$ $2 \cdot T_{1m}$ }, where T_{1m} and T_{2m} denote the central value (e.g., average or median) of the first and second mode period, respectively, of the index building set. For use with lowrise buildings, where the second mode is not influential, only three T_i values are needed: { T_{1m} , $1.5 \cdot T_{1m}$, $2 \cdot T_{1m}$ }, Here, each measure of damped elastic spectral acceleration response S_a is the geometric mean of two orthogonal directions. For the default intensity measure types, evaluate structural response at the following intensity measure levels:

 $T_{1m} \le 0.5 \text{ sec: } x \in \{0.1g, 0.2g, 0.3g, \dots 3.5g\}$

 $0.5 < T_{1m} \le 2 \text{ sec: } x \in \{0.1g, 0.2g, 0.3g, \dots 1.5g\} > 0.5$

 $0.2 \text{ sec} < T_{1m}$: x $\in \{0.1g, 0.2g, 0.3g, \dots 0.5g\}$

If an index building is too computationally demanding to analyse at this many levels of x, the analyst can use

 $x \in \{10^{-1.00}, 10^{-0.75}, 10^{-0.50}, \dots 10^{0.50}g\}$. This requires analysis at a maximum of 7 intensity measure levels.

3.4.2 Option 1: simplified structural analysis

This method tends to over-predict acceleration response at the top of the building and to underpredict acceleration response at the bottom of the building, because it neglects higher modes. These errors should tend to cancel each other out when summing up the costs of different stories. Therefore, for simplicity, let

 T_{1m} = mean fundamental period of vibration, sec., of the index buildings

$$T_{1m} = \frac{1}{n} \sum_{i=1}^{n} T_{1,i}$$
(18)

 $T_{1,i}$ = estimated fundamental period of vibration of index building *i*, from structural analysis, local guidelines, or use the following empirical (median) default values:

= 0.0905Z^{0.8} steel moment-resisting frame (Chopra and Goel 2000)

= $0.0524Z^{0.9}$ concrete moment-resisting frame (Chopra and Goel 2000)

= 0.0975*Z*^{0.75} steel eccentrically braced frame or steel buckling-restrained braced frame (Tremblay 2005)

= $0.0610Z^{0.75}$ all others (ASCE 7-10, increased by a factor of 1.25 to eliminate conservatism)

Z = building height, meters

Let $s_{ha}(x)$ denote the story-level peak horizontal acceleration at floor *h* given that $S_a(T_{1m}, 5\%)$ takes on a value of *x*. It can be calculated during the structural analysis, or approximated as follows:

$$s_{ha}(x) = PGA + \varphi(h) \cdot \left(\Gamma \cdot S_a(T_{1m}, 5\%) - PGA\right)$$

$$\leq S_{max}$$
(19)

$$s_{hd}(x) = \Gamma \frac{S_a(T_{1m}, 5\%)T_{1m}^2}{4\pi^2} \cdot \frac{(\varphi(h+1) - \varphi(h))}{z_{h+1} - z_h}$$
(20)

Where

PGA = peak ground acceleration in units of g

 \approx S_a(1 sec, 5%) for T_{1m} ≥ 0.5 sec

 $\approx 0.4 \cdot S_a(0.3~\text{sec},\,5\%)$ for $T_{1\text{m}} < 0.5~\text{sec}$

 $\phi(h,x)$ = response at floor h, normalized by response at the roof, given x. Evaluate it at each value of x

through structural analysis. Alternatively, use the following default for all values of x as follows:

$$\varphi(h) = \frac{z^2}{Z^5} \cdot \frac{\left(70Z^3 - 40Z^2z + 5Zz^2 + 2z^3\right)}{37} \quad \text{shearwall building} \tag{21}$$

$$\varphi(h) = \frac{z}{Z^3} \cdot \frac{(12Z^2 - 3Zz - 2z^2)}{7}$$
 frame building (22)
$$\varphi(h) = \frac{z}{Z}$$
 dual system (23)

(These default mode shapes for shearwall and frame buildings are derived from first principles using the deflected shape of a cantilever beam subjected to linearly increasing (triangular) distributed load. For a shearwall building, the beam is taken as having infinite shear modulus (GA = ∞) and finite bending stiffness (EI < ∞). For a frame building, the beam is taken as having infinite shear modulus (GA = ∞) and finite shear modulus (GA = ∞). The default mode shape for a dual system is taken as linear.)

 Γ = roof acceleration as a factor of modal acceleration

≈ 1.3

S_{max} = maximum strength of the fully-yielded system normalized by the effective seismic weight, *W*, of the structure, i.e., the y-value of the building's pushover curve at ultimate in spectral coordinates.

z = height of story h above the ground, meters

Z = roof height, meters above the ground

Calculate $s_{ha}(x)$ for each story *h* at each level of *x*.

Also calculate the collapse probability at each level of x, given by

$$P_{c}(x) = \Phi\left(\frac{\ln\left(x/\hat{S}_{CT}\right)}{\beta_{TOT}}\right)$$
(24)

where

 \hat{S}_{CT} = median collapse capacity of the building in terms of S_a(T,5%) and β_{TOT} denotes the total logarithmic standard deviation of the collapse capacity. The user can calculate both values by methods specified in D'Ayala et al. (2015), or by FEMA P-695, or by the following simplified method based on FEMA P-695, illustrated in Figure 2:

$$\hat{S}_{CT} = C_s \cdot 1.5 \cdot R \cdot CMR \cdot SSF$$
(25)

$$\beta_{TOT} = 0.8 \tag{26}$$

 C_S = seismic response coefficient (see below for example of how to calculate C_S in the United States). Note that it is defined as $C_S = V/W$, i.e., design base shear as a fraction of building weight, but in the US is calculated as shown later. It may be calculated in other ways in other countries.

- V = design base shear, units of force
- W = building weight
- R = response modification factor, essential ductility demand at design-level ground motion. Can be taken from ASCE 7-10 Table 12.2-1 (duplicated in Appendix C) or from local standards.
- *CMR* = collapse margin ratio, as defined in FEMA P-695: "The ratio of the median 5%-damped spectral acceleration of the collapse level ground motions, \hat{S}_{CT} (or corresponding displacement, SD_{CT}), to the 5%-damped spectral acceleration of the MCE ground motions, S_{MT} (or corresponding displacement, SD_{MT}), at

the fundamental period of the seismic-force-resisting system." In the US, ordinary buildings are designed to resist base shear of $2/3 \cdot S_{MT}/(R/I_e)$, where I_e is an importance factor, generally though not always 1.0, and R is as defined above. Default values for CMR:

- = 1.0 for unreinforced masonry or earthen structure
- = 1.5 for special reinforced concrete moment frame
- = 2.0 for others

SSF = spectral shape factor, as defined in FEMA P-695. Default value = 1.15



Figure 2. Relating median collapse capacity to the pushover curve (FEMA P-695, Applied Technology Council 2009)

For example, in the U.S., one could calculate C_s from ASCE 7-10 Sec 12.8.1.1, as follows:

$$C_{S} = \frac{S_{DS}}{\left(\frac{R}{I_{e}}\right)}$$
(27)

where, per ASCE 7-10,

- S_{DS} = design, 5 percent damped, spectral response acceleration parameter at short periods as further defined in ASCE 7-10 Section 11.4.4. For the reader unfamiliar with ASCE 7-10's procedures, SDS is calculated as in Equation (28)
- R = response modification coefficient as given in ASCE 7-10 Tables 12.2-1, 12.14-1, 15.4-1, or 15.4-2

 α = importance factor as prescribed in ASCE 7-10 Section 11.5.1

$$S_{DS} = \frac{2}{3} F_a S_S \tag{28}$$

where

- F_a = short-period site coefficient (at 0.2 s-period); per ASCE 7-10 Section 11.4.3.
- $S_{\rm S}$ = mapped risk-targeted maximum considered earthquake (MCE_R) 5 percent damped, spectral response acceleration parameter at short periods as defined in ASCE 7-10 Section 11.4.1.

3.4.3 Option 2: nonlinear pseudostatic (pushover) analysis

In general, methods based on nonlinear pseudostatic procedures are not advisable for highrise structures. The errors in estimating interstory drift ratios (rather than the global roof drift) rise quickly with increasing height and with the contribution of higher modes. Advanced procedures have appeared in recent works that incorporate multiple modes to improve upon the error of standard single-mode pushover methods. Since structural modelling is typically the most labor-intensive part of the structural analysis, and given that all such improved static procedures are quite complex and unsupported by commercial software, they are not recommended at this point in time. If higher accuracy is sought then, unless the analyst possesses the necessary skills to run such methods, the analyst should resort to option 3, nonlinear dynamic analysis.



Figure 3. The proposed nonlinear static pushover based method (option 2) as shown in coordinates of the intensity measure versus response (typically used for incremental dynamic analysis). MRSA results, normally applicable only to the elastic region are extended beyond the yield point in accordance with the equal displacement rule for moderate and long period structures. SPO2IDA provides the upper limit of this extension in terms of the intensity measure, showing where collapse is expected to appear (in a median/mean sense). In terms of incremental dynamic analysis, this provides the "flatline" where response becomes (nominally) infinite.

Still, the classic single-mode pushover analysis has been shown to be a viable approach for estimating collapse capacity (Vamvatsikos and Cornell, 2005b), especially if some care is exercised to find the pushover curve that best describes the path followed by the structure to collapse. The present guideline employs this approach for pushover analysis to evaluate global collapse. Elastic modal response spectrum analysis (MRSA) is employed to evaluate structural response given that no collapse has occurred (the portion of the pushover curve in Figure 3 up to the collapse point). In the case of a lowrise building, MRSA should be replaced by a single-mode pushover method, e.g., as described in EN 1998-3 (Comité Européen de Normalisation, 2005) or ASCE 41-06 (American Society of Civil Engineers 2007).

The proposed pushover-based analysis for highrise buildings is as follows:

Step 1: Select a representative hazard spectrum for the analysis. One could employ a uniform hazard spectrum or a design spectrum. These tend to bias the estimate of median response. This guideline recommends using the median spectrum from 7 or more ground motion records, such as the 22 pairs of (unscaled) horizontal components from far field recordings employed by the authors of FEMA P-695 (see Applied Technology Council, 2009, Appendix A). The ground motions are currently available at http://www.csuchico.edu/structural/researchdatabases/ground_motion_sets.shtml.

- Step 2: Perform MRSA to estimate response values for interstory drifts and peak floor accelerations (see Box 4). Estimate the level of the intensity measure corresponding to the adopted spectrum. The IM can be any of the single period $S_a(T)$'s listed in Section 3.4.1. $S_a(T_{1m})$ is the recommended option, where T_{1m} is the mean T_1 of all index buildings. To estimate the median response for any other level of the IM, linearly scale the results found for the present value of $S_a(T)$.
- Step 3: Perform a nonlinear static pushover using a first-mode load pattern (or a load pattern of, e.g., parabolic shape that incorporates higher modes) until the maximum strength is reached. Then continue pushing either with (a) the original pattern or (b) a uniform or (c) an inverse triangular (the original pattern turned upside down). Adopt whichever pushover curve is the most aggressive (lower strength and lesser ductility) of the three. Apply the SPO2IDA tool (https://www.atcouncil.org/projects/ATC-58-1/FEMAP-58-3_Tools.zip) to get the median and dispersion of collapse capacity. For each index building supply the building's own T_1 to SPO2IDA. Then use the spectrum of step 1 to find the ratio of S_a at the period adopted for the IM (T_{1m} being the recommended) over S_{α} at period T_1 . Use this ratio to convert (by simple multiplication) the resulting median $S_o(T_1)$ value of collapse to the period of the common IM. Note that while SPO2IDA also provides comprehensive roof drift information at all levels of intensity, these only correspond to the first mode and are, thus, not considered representative of a tall structure.
- Step 4 (optional): If using only three index buildings, proceed with the supplied information: median responses at each IM level and median and logarithmic standard deviation of collapse capacity. If using seven index buildings, then the logarithmic standard deviation of response at each IM level, given that no collapse has happened, is also required. For interstory drifts, assume a uniform 10% initial logarithmic standard deviation for any story drift given the IM level up to the (nominal) yield value of S_a (roughly estimated as the yield strength from the static pushover divided by the weight) and then linearly interpolate between this yield value and the median collapse IM, assigning the logarithmic standard deviation of collapse capacity at the median collapse IM. For peak floor accelerations, similarly interpolate by assigning 1.3 times the logarithmic standard deviation of collapse capacity at yield and a logarithmic standard deviation of 0.3 at collapse. (Peak floor accelerations saturate after yield, thus losing variability.) To generate discrete values similar to the results of nonlinear dynamic analyses from different ground motion records (needed to apply Monte Carlo for uncertainty propagation) that can represent the dispersion of each response EDP given the IM value, simple stratified sampling can be employed. Assuming a lognormal distribution, it is sufficient to get 7 such discrete points for each response type (each peak floor acceleration and story drift) for each level of IM. To do so, evaluate $DP_i(x) = m_{dp}(x) \cdot exp[\mathcal{B}_{dp}(x) \cdot \mathcal{K}_i]$ where $m_{dp}(x)$ and $\mathcal{B}_{dp}(x)$ are the median and logarithmic standard deviation of the demand parameter at intensity measure level x, while the standard variates are K_i = -1.4652, -0.7916, -0.3661, 0.0, 0.3661, 0.7916, 1.4652, for i =1,2,...,7.

The proposed pushover-based analysis for low- and midrise buildings is as follows:

- Step 1: Same as for highrise buildings
- Step 2: Perform a nonlinear static pushover analysis using a first-mode load pattern. Informed analysts may employ alternative nonlinear static procedures at will (e.g., adaptive pushover, different load patterns deemed more suitable to a given structure etc.). For some classes of buildings there are also

methods such as DBELA (Calvi 1999, Crowley et al. 2004) and FaMIVE (D'Ayala and Speranza 2003) for directly defining the pushover curve and all pertinent drift responses without performing an actual analysis. Regardless of the method employed, the IM can be any of the single period $S_a(T)$'s listed in Section 3.4.1. $S_a(T_{1m},5\%)$ is the recommended option, where T_{1m} is the mean T_1 of all index buildings. Define a range of equally spaced values for the IM of choice, e.g., at steps of 0.1g. Then, use the spectrum of step 1 to find the ratio of S_a at T_1 (i.e. the analysed building's own fundamental period) over S_a at the period adopted for the IM. Use this ratio to convert (by simple multiplication) levels of the common IM to $S_a(T_1.5\%)$ values for the specific building. Employ either EN 1998-3 (Comité Européen de Normalisation 2005), ASCE 41-06 (American Society of Civil Engineers 2007) or the SPO2IDATool (Vamvatsikos 2010) to transform $S_a(T_1, 5\%)$ values, into strength ratio R values, then to roof displacements (or equivalent SDOF displacements) and finally to each story's interstory drifts. Estimate corresponding peak floor acceleration values according to Box 4. In the end, the analyst should have the interstory drift and peak floor acceleration values for each story and at each level of the common IM adopted for the set of index buildings.

Step 3: Apply the SPO2IDATool (Vamvatsikos 2010) to get the median and dispersion of collapse capacity. For each index building supply the building's own T_1 to SPO2IDA. Use the ratio of IM over $S_a(T_1)$ (i.e. the inverse of the ratio derived in the previous step for the selected spectrum) to convert the resulting median $S_a(T_1)$ value of collapse to the common IM period (by simple multiplication). If SPO2IDA has also been used in the previous step, then step 2 and step 3 actually become merged.

Step 4 (optional): Same as for highrise buildings.

Box 4: Multi-modal estimation of peak floor accelerations

One must estimate absolute peak floor acceleration (PFA) at each floor of each index building with acceleration-sensitive components. When employing a pushover-based method, estimation of PFAs is sensitive to higher modes, even when their effective mass is relatively low. Thus, it is recommended to use the following simplifying method that uses a square-root-sum-of-squares rule over all *N* modes (or at least those accounting for a minimum 98% of the total mass) to approximate PFAs at each floor. Then, the elastic PFA demand in story *s* can be estimated by applying the square-root-sum-of-squares (SRSS) rule to the pseudo acceleration response spectrum.

$$PFA_s \approx \sqrt{PGA^2c_s^2 + \sum_{j=1}^N \left(\Gamma_j \phi_{j,s} S_a(T_j,\xi)\right)^2 + 2PGAc_s \sum_{j=1}^N \left(\Gamma_j \phi_{j,s} S_a(T_j,\xi)\right)}$$

Where c_s rules the spatial contribution of PGA:

$$c_s = 1 - \sum_{j=1}^N \left(\Gamma_j \, \phi_{j,s} \right)$$

The above yields $PFA_0 = PGA$ at the ground floor. Γ_j is the modal participation factor for mode j and $\varphi_{j,s}$ is its corresponding component for story s. $S_a(T_j, \zeta)$ is the spectral acceleration at mode period T_j and damping ratio ζ for elastic response. In other words, for any point along the pushover where the strength ratio $R \le 1$. For higher values of R, i.e. responses past the nominal yield point of the capacity curve, the PFA values at R = 1 should be used. To estimate the PGA and S_a values, the elastic response spectrum used for MRSA should be

employed. Note that the main assumption in the above equation is that the frequencies are uncorrelated and furthermore the frequencies are uncorrelated with respect to PGA. This is only true for stories well above the ground level. Thus, some unconservative bias reaching up to 30% can be expected, a fact that is reflected in the value of the β_m factor (Eq. 6) for adding uncertainty at the component level. An extended CQC rule according to Taghavi and Miranda (2008) can significantly improve the results at the expense of more complex calculations.

Box 5: General guidance on structural modeling

Typically, a considerable part of the effort in vulnerability estimation lies with creating a structural model of the index buildings. In this respect, it is often advantageous to adopt a reduced MDOF model employing story-level (rather than component-level) characterization of mass, stiffness and strength. One estimates story-level characteristics rather than creating a 2D or 3D component-by-component structural model. Equally importantly, such models can be analysed within a few seconds using either nonlinear dynamic or static methods. Two such types of models are presented together with the option of a conventional detailed model.

A. Stick models

The basic idea stems from the use of fishbone models (Luco et al. 2003, Nakashima et al. 2002) to represent moment-resisting frame buildings using only a single column-line with rotational restrictions for each floor owing to the presence of beams. This idea is hereby simplified to cover many different structural systems as shown in Figure 4. It comprises *N* nodes for *N* stories, each with 3 degrees of freedom (horizontal, vertical, rotational) in 2D space. The nodes are connected by *N* columns in series and further restrained rotationally by *N* springs representing the strength and stiffness of beams at each floor. All elements are nonlinear, at the minimum having a simple elastic-perfectly-plastic force-deformation (or moment-rotation) behavior with an ultimate (capping) ductility limit (or a dramatic loss of strength) that is explicitly modeled. Element characteristics can be derived using the aggregate stiffness of the columns, piers, walls. and beams in each story, together with the corresponding yield and ultimate displacements or rotations.

Columns may be modeled using lumped-plasticity or distributed-plasticity force-based beam-column elements. Displacement-based elements are not recommended unless every column is represented by at least four such elements, with the ones closer to the ends being considerably smaller to allow for a reliable localization of deformation. In all cases, for each story-level column the analyst should define the moment-rotation characteristics of the element section. Assuming a capped elastic-plastic model (Figure 5) at the very minimum, together with a lumped-plasticity column representation, each story of a given height is characterized at minimum by the following parameters:

(1) Story column plastic hinge, taken to represent the total stiffness and strength of all the columns in the story. The yield and ultimate rotations need to represent the average values across all story columns.

- (2) Beam rotational spring, to represent the total stiffness of N_b beams in double curvature bending, the strength of $2N_b$ beam rotational hinges and the average of the corresponding rotational ductilities (including any slab contributions, if thought to be significant).
- (3) The story translational mass, to be applied at each story node

P-Delta effects are taken into account by applying appropriate gravity loads and assigning the proper geometric transformation to columns. For use with perimeter (rather than space) frame systems, the use of a leaning column is not necessary as the entirety of the story mass (and gravity load) is applied at the single story node. Still, this means that the area of the column element, but not its moment of inertia, needs to be increased to represent the total column are of both moment-resisting and gravity framing elements.



Figure 4. A three-story stick model, showing rotational beam-springs, column elements and floor masses M₁ – M₃





In general, stick models are not recommended for cases where the building height is larger than three times its width, as the flexural component of deformation due to column elongation may become important. Additionally, they are only appropriate for structures having rigid diaphragms without appreciable plan irregularities. When dual systems are to be modeled, e.g. systems where both structural walls or braces and

moment-frames significantly contribute to lateral stiffness and strength, it is advised to employ two stick models side-by-side, connected by horizontal translation constraints to represent the rigid diaphragm.

B. Single-bay frame

Whenever it is desirable to further distinguish the behavior of the column springs to their individual constituents, a single-bay multi-story frame may be employed instead of a simple stick. Each story is now represented by two columns and one connecting beam plus any additional element acting at the story level, such as braces and infill walls. While a complex story-level element would need to be defined for an equivalent stick model, the larger number of elements employed by the single-bay model allows an easier way for defining the behavior of the story. For example, braces and infill walls can be explicitly added by the inclusion of the proper element(s), as for a proper 2D component-by-component model. Definition of beam and column characteristics follows exactly the details laid out for the stick model. The only difference is that all strength and stiffness terms need to be divided equally among the two columns. Similarly, each story's beam has now two distinct plastic hinges. Thus each of those should represent the contribution of N_b beam plastic hinges, where N_b is the number of bays. The same restrictions of use should be observed as for stick-models, although, extended applicability to higher aspect ratios is expected.

C. Detailed 2D/3D models

A proper 2D or 3D model of the full-scale structure needs to be created following either to a great or a lesser detail, making sure that the most important features of the class are retained. The choice of a 2D or 3D model depends on the plan asymmetry characteristics of the building, 3D being most appropriate wherever significant eccentricity (torsion effects) are expected. Appropriate representation of the nonlinear behavior of lateral-load resisting elements and any inherent global or local geometric nonlinearity (e.g., P-Delta effects or brace buckling) is essential. For more details, see Deierlein et al. (2010). It is noted that only engineers well versed in nonlinear structural modeling should follow this approach for vulnerability analysis. It is bound to be quite time-consuming for the average user.

3.4.4 Option 3: nonlinear dynamic structural analysis

Guidance in this section focuses on selecting ground-motion time histories for use in the structural analysis, and then extracting the structural response measures discussed in Section 3.4.1. See Box 5 for general guidance on creating the structural model.

We recommend using incremental dynamic analysis (IDA) for performing the nonlinear dynamic analyses. The analysis produces the conditional distribution of each demand parameter DP conditioned on the intensity measure IM for each of many IM levels, from elasticity to global collapse. The structural model is subjected to nonlinear dynamic structural analysis under a suite of ground-motion accelerograms that are scaled to increasing levels of the IM until collapse is reached. To achieve good assessment resolution, at least 18 ground motion recordings should be employed. Ground motions are available from a number of sources, including FEMA P-695 (far-field set), <u>http://peer.berkeley.edu/products/strong_ground_motion_db.html</u>,

and <u>www.cosmos-eq.org</u>. These last two are appropriate for shallow crustal earthquakes in active tectonic regimes anywhere in the world. For a 3D model this should become at least 18 pairs of horizontal components, each pair recorded at the same station and event. For a 2D model this translates to using at least 18 horizontal components, none of which belongs to the same recording. In other words, for 2D analysis only a single component of each pair is used, although it is highly desirable to have both components available to allow the estimation of the geometric mean of spectral acceleration of both components at any given period. If it is considered to be of interest and the model supports its use, the vertical ground motion component may also be added to the corresponding horizontal one(s). As of this writing, the FEMA P-695 farfield ground motions are available at http://www.csuchico.edu/structural/researchdatabases/ground_motion_sets.shtml.

For most general cases, the 22 pairs of ground motions comprising the FEMA P-695 far-field ground motion set can be employed as a standard set. As IDA uses only scaling to modify the ground motions, it functions best when far field records are used together with an efficient and sufficient IM. Far-field records contain no traces of directivity (i.e. near source pulses). Typically, for use with vulnerability assessment, instances of soft soil amplification should also be avoided, unless the analyst knows this to be the dominant case. As for the IM, use of $S_{agm}(T_i)$, i.e. the geometric mean of both horizontal components of ground motion at several periods of interest, is considered to be a good choice that significantly reduces the variance and the bias typically associated with large scale factors. Use of a single-period $S_a(T_{1m})$ (also as a geometric mean of both horizontal components) instead should only be considered in cases of strong ground motions and relatively weak buildings where scale factors higher than 3.0 are not expected. Regardless of the choice, contrary to the usual practice when running IDA for a single building, it is important herein to use the same IM for all index buildings to avoid having to do some complex postprocessing later on. See also the relevant discussion in Section 4.4 regarding the selection of ground motions and IMs for additional information.

There are efficient approaches to running IDA, for example the hunt-and-fill algorithm by Vamvatsikos and Cornell (2002) that employs 7-9 runs per record to achieve good results with high computational efficiency. These need some additional postprocessing, though, to derive the necessary distributions of DP conditioned on IM. For analysts familiar with Matlab and OpenSees, a full suite for efficiently running and post-processing IDA results is provided by <u>http://users.ntua.gr/divamva/software.html</u>. A less efficient but simpler algorithm is shown below:

- 1) Select the constant IM-step, say $\Delta x = 0.1g$ for T \leq 1sec, $\Delta x = 0.2g$ for T > 1sec (or as suggested in the discussion of IM choices)
- 2) Increment IM by the IM step, starting from 0.
- 3) For each record that has not registered a collapse at an earlier IM, perform the time-history analysis and keep only the following information: (a) whether collapse has occurred, and if not then (b) the peak absolute values of absolute peak floor acceleration (PFA) and peak transient interstory drift (PTD) at each story.

Interpret any of the following conditions as collapse:

(a) The structural analysis does not converge. This is not to be confused with non-convergence caused by ill-prepared models or badly run timehistory analyses. Small time steps and good modelling practices are important.

- (b) The peak transient interstory drift ratio on any story exceeds a reasonably high value. For all but very ductile building such as woodframe shearwall buildings, 10% peak transient drift ratio is a reasonably high value to use as a proxy for collapse. For woodframe shearwall construction, 20% peak transient drift can be reasonably assumed to result in collapse. Attention should be paid to make sure that the model has been adequately supplied with details (see minimum modelling requirements discussed earlier) that can help it properly simulate collapse. If ductility limits are not included for all non-linear components of the structure, then this 10% value becomes a completely artificial number to determine collapse for a model that cannot predict it. Its use is acceptable only for realistic models that do show a loss of strength due to P-Delta or in-cycle degradation but simply happen to stay numerically stable up to large drift values.
- (c) A non-simulated collapse mode has occurred, e.g. shear failure of a column or joint.

Care should be exercised in cases where records may show non-collapsing response even above the IM level where first collapse was recorded (a phenomenon called structural resurrection). Such results should be ignored and only the point of first collapse should be considered.

At each level of IM, the probabilistic characterization of structural response is achieved by (a) the collapse probability denoted here by $P_c(x)$ equal to the number of records that have reached collapse (N_c) divided by the total number of records and (b) the distribution of responses given x (formally DP|IM) supplied by the non-collapsing records for each demand parameter of interest (peak interstory drift ratios and peak floor accelerations). The latter can be directly represented by the actual DP values registered by each non-collapsing record or by their statistical characterization, e.g. their median and log standard deviation, assuming they are lognormally distributed. In the present work, we recommend the former, as lognormality may or may not be appropriate.

3.4.5 Alternative methods to estimate collapse fragility

There are other approaches to estimating collapse fragility. For example, FEMA P-58 (Applied Technology Council 2012) offers a method to estimate judgment-based collapse fragility. Again, as with the rest of the structural analysis, the analyst is free to estimate collapse fragility by any method that is convenient, familiar, and deemed professionally sound by the project team, as long as (1) it estimates the collapse probability at each of the intensity measure levels of interest, and (2) the collapse-fragility procedure—its strengths and weaknesses and the reasons the analyst selected it—is disclosed to the consumer of the vulnerability functions.

3.5 Step 4: Derive building-level vulnerability functions

Finally, calculate the expected value of damage factor at each value of x as follows, where RCN is the replacement cost (new) of the building, N_h is the number of stories, and f_1 is the fraction of total building replacement cost (new) represented by components in the inventory. If the analyst only wants a vulnerability function for non-structural components, then f_1 is the fraction of the total replacement cost (new) of the non-structural components by the components in the inventory. For simplified structural analysis,

$$y(x) = P_c(x) + (1 - P_c(x)) \cdot \frac{E[C|S = s(x), NC]}{RCN}$$
⁽²⁹⁾

$$E\left[C\left|S=s\left(x\right),NC\right]=\frac{1}{f_{1}}\cdot\sum_{h=1}^{N_{h}}\left(E\left[C_{a}\left|S_{h,a}=s_{h,a}\left(x\right)\right]+E\left[C_{d}\left|S_{h,d}=s_{h,d}\left(x\right)\right]\right) if \le 0.6$$

$$=1.0 \ otherwise$$
(30)

For nonlinear dynamic structural analysis or user-selected analyses that require multiple structural analyses per intensity measure level,

$$y(x) = P_c(x) + (1 - P_c(x)) \cdot \frac{E[C|S = s(x), NC]}{RCN}$$
(31)

$$E\left[C\left|S=s(x),NC\right]=\cdot\frac{1}{\left(f_{1}\cdot n^{*}(x)\right)}\cdot\sum_{m=1}^{n^{*}(x)}\sum_{h=1}^{N}\left(E\left[C_{a}\left|S_{h,a}=s_{h,a,m}(x)\right]+E\left[C_{d}\left|S_{h,d}=s_{h,d,m}(x)\right]\right)\right) if \le 0.6$$

$$=1.0 \ otherwise$$
(32)

where $n^*(x)$ is the number of ground motion pairs that did not result in collapse at intensity measure level x, $s_{h,a,m}(x)$ is the geometric mean floor acceleration at floor h in ground motion pair m (excluding cases of collapse), and $s_{h,d,m}(x)$ is the peak transient drift ration at story h in ground motion pair m (excluding cases of collapse). When repair cost exceeds 0.6, the building is commonly considered a total loss, hence the jump to a damage factor of 1.0 when the repair cost exceeds 0.6 constructive total loss, meaning that a partial loss has occurred to an extent that the property is beyond economic repair.

If the analyst's objective is to estimate the vulnerability function for fatalities, then instead of Equations (29) through (32), use:

$$y(x) = P_c(x) \cdot \frac{(f_L + f_H)}{2}$$
(33)

where f_L and f_H are respectively So's (2012) lower and upper recommended fatality rate for the building class under consideration. In Equation (33), y(x) denotes the expected value of fatality rate at intensity measure level x. They are repeated here for convenience in Table 6. So offers separate fatality-rate ranges for rare cases of what she calls "catastrophic collapse," like pancake collapse. She acknowledges that although these figures are "difficult to incorporate in general loss models," they can "inform our understanding of the survivability of occupants in such buildings." We repeat them, as she offers them, for information purposes. If the analyst has strong reason to believe that the buildings in question are subject to pancake collapse, we suggest using f_L and f_H for the typical (non-pancake) case and then offering results using the catastrophic collapse figures as a possible outcome, even if the analyst cannot provide a probability of catastrophic collapse.

Table 6. So (2012) judgment-based fatality rates in collapsed buildings for implementation in earthquake loss

octi	ma	τιΛ	nı	mn	ao	IC
csu	IIIa	ιυ		110	uc	13

Row	Building class	fL	fн
1	Light timber with light roof	0%	1%
2	Light timber with heavy roof	1%	3%
3	Heavy timber with light roof	1%	1%
4	Heavy timber with heavy roof	2%	3%
5	Weak masonry adobe light roof	5%	15%
6	Weak masonry adobe heavy roof	20%	90%
7	Weak masonry irregular stone with wooden pitched roofs lowrise	5%	20%

Row	Building class	fL	f _H
8	Weak masonry irregular stone low-rise concrete slab roofs	10%	40%
9	Load-bearing masonry, European	3%	12%
10	Load-bearing masonry, Asian	10%	25%
11	Reinforced masonry lowrise	2%	8%
12	Reinforced masonry midrise	15%	40%
13	Confined masonry	2%	6%
14	Confined masonry with pancake collapse	10%	40%
15	Mixed confined masonry and unreinforced masonry	15%	20%
16	Concrete frame no code lowrise	10%	15%
17	Concrete frame no code midrise	15%	30%
18	Concrete frame no code highrise	40%	60%
19	Concrete frame low code lowrise	5%	15%
20	Concrete frame low code midrise	10%	25%
21	Concrete frame low code highrise	30%	50%
22	Concrete shearwall lowrise	5%	15%
23	Concrete shearwall midrise	10%	20%
24	Concrete shearwall highrise	10%	25%
25	Concrete shearwall lowrise with catastrophic collapse	40%	60%
26	Concrete shearwall midrise with catastrophic collapse	50%	70%
27	Concrete shearwall highrise with catastrophic collapse	60%	80%
28	Steel frame lowrise	10%	15%
29	Steel frame midrise	15%	25%
30	Steel frame highrise	20%	25%
31	Steel braced frame with catastrophic collapse	50%	70%
32	Light metal frame	4%	6%

3.6 Step 5: Mean vulnerability function and uncertainty

3.6.1 Uncertainty

It is usually desirable to estimate uncertainty in the vulnerability function. It is quantified here for repair cost using the coefficient of variation of loss at each value of x. Let us denote it by v(x). (We do not consider uncertainty in fatality rate, since the necessary quantities about uncertainty in fatality rate conditioned on collapse are not yet available.) We treat v(x) here as having two distinct contributions:

- 1. Building-to-building variability. This is the uncertainty in loss arising from the fact that different buildings of the same class will perform differently when subjected to the same level of IM. Different buildings of a given class may vary in terms of number of stories, plan and elevation shape, number of bays, material properties, design strength and stiffness, differences in construction quality, and other features. One can estimate the building-to-building variability using the results of both the 3-and 7-index-building options.
- 2. Within-building variability. This is the uncertainty in loss at a given level of IM for a particular building, because of (a) record-to-record variability in structural response at a given level of IM, (b) uncertain component capacity, that is, the uncertain level of structural response that will produce prescribed damage states, (c) uncertain unit repair costs, that is, the uncertain cost to repair each component of a given type from a specified damage state, (d) uncertainty in material properties not reflected in the deterministic structural model and (e) other modelling errors, that is, the difference

in structural response between how the real building in nature behaves and its how the mathematical idealization estimates its behavior. This guideline document provides procedures for calculating within-building variability only for the 7-index-building option.

3.6.2 One or three index buildings

If the analyst considered one index building, then the mean vulnerability function for the class is given by the vulnerability function for the one index building, from Equation (31).

If the analyst considered three index buildings (poor, typical, and superior), then the mean vulnerability function for the class is given by the simple average of the three, as shown in Equation (34), where $y_k(x)$ for each index building k (k = 1, 2, and 3) is given by Equation (31).

$$y(x) = \frac{1}{3} \sum_{k=1}^{3} y_k(x)$$
(34)

If the analyst chose to use a single index building, then we recommend using the function suggested by Porter (2010):

$$v(x) = \frac{0.25}{\sqrt{y(x)}} \tag{35}$$

This curve was regressed from the implied coefficient of variation of damage factor from all HAZUS-MH model building types—old and new, weak and strong, stiff and flexible, brittle and ductile—which jointly exhibit a fairly consistent relationship between v(x) and y(x). Note that Equation (35) will tend to overestimate uncertainty as collapse comes to dominate loss.

If the analyst chooses to assess three index buildings, then one can explicitly calculate the between-building, within-class coefficient of variation of damage factor conditioned on mean damage factor because one has three distinct mean vulnerability functions to draw upon. Within-building variability is not explicitly calculated for the three-building approach. To estimate it, one assumes the building-to-building and within-building uncertainties are the equal at a given intensity measure level. Under this assumption, one can calculate the coefficient of variation as follows.

$$v(x) = 1.4 \frac{\sqrt{\frac{1}{2} \sum_{k=1}^{3} (y_k(x) - y(x))^2}}{y(x)}$$
(36)

The approximate equivalence of building-to-building and within-building uncertainty is supported by the observed ratio of v(x) for all HAZUS-MH building classes and the analytically derived v(x) from the CUREE-Caltech Woodframe Project (Porter et al. 2002). See Appendix D.2 for details.

Once one has calculated y(x) and v(x), the conditional distribution of loss can be taken as lognormal with mean and coefficient of variation as described above (truncating at 1.0), or as beta with bounds 0 and 1 and the same mean and coefficient of variation.

3.6.3 Seven index buildings

If the analyst considered seven index buildings, then one has the data necessary to explicitly calculate mean vulnerability, within-building variability, and between-building variability. In appendix B.2, we provide guidance on Monte Carlo simulation of repair costs. Each simulation produces a simulated value of the damage factor, denoted here by $Y_{k,q,sim}(x)$, where Y denotes uncertain damage factor, k denotes the index building number, q denotes an index to quality level (q = 1 denotes generally poor quality components, q = 2 denotes a typical mix, and q = 3 denotes a generally superior or rugged mix of components), *sim* denotes an index to the simulation number, and x denotes the intensity measure level.

To be clear about quality levels: we refer here to 7 index buildings, and each index building is represented by a single structural model. But each index building is provided with three variants: poor, typical, and superior, for a total of 21 variants. The structural analysis is performed at the index-building level, and assumes that component quality does not affect structural response. The damage and loss analysis takes place at the variant level, after the structural analysis, and where component quality does affect loss. $Y_{k,q,sim}(x)$ is calculated as follows

where

RCN = replacement cost new. See Appendix B.2 for notes on simulating RCN.

 f_1 = inventory construction cost as fraction of RCN

h = index to floor level (in the first sum) or story level (in the second sum). "Floor" refers to the ground level, a floor level, or roof level, where h = 0 refers to the ground floor. "Story" refers to the space between two floors. where h = 1 refers to the ground story.

 N_h = number of stories

- a = an index to acceleration-sensitive components
- r = an index to drift-sensitive components
- N_a = number of acceleration-sensitive component types in the inventory
- N_r = number of drift-sensitive component types in the inventory
- d = index to damage state
- $N_{a,d}$ = number of possible damage states of acceleration-sensitive component type a
- $N_{r,d}$ = number of possible damage states of drift-sensitive component type r
- $n_{h,a,d}$ = number of acceleration-sensitive components on floor h of type *a* in damage state *d*. See Appendix B.2 for notes on simulating $n_{h,a,d}$.
- $n_{h,r,d}$ = number of drift-sensitive components on floor h of type r in damage state d. See Appendix B.2 for notes on simulating $n_{h,r,d}$.

- $u_{a,d}$ = unit cost to repair acceleration-sensitive components of type *a* from damage state *d*. See Appendix B.2 for notes on simulating $u_{a,d}$.
- $u_{r,d}$ = unit cost to repair drift-sensitive components of type *r* from damage state *d*. See Appendix B.2 for notes on simulating $u_{r,d}$.
- Y_{CTL} = uncertain damage factor associated with constructive total loss. This is the damage factor above which the building is treated as a total loss. Absent better information, assume Y_{CTL} is normally distributed with mean 0.6 and coefficient of variation 0.1.

Note that Equation (37) gives the damage factor in simulation k considering the possibility of non-collapse or collapse, while Equation (38) adjusts this amount to account for the fact that buildings are sometimes treated as a total loss when the repair cost exceeds a threshold quantity referred to as constructive total loss, even if that quantity is less than the replacement cost new of the building.

The mean and standard deviation of the vulnerability function at excitation x for index building k is given by

$$y_{k}(x) = \frac{1}{3 \cdot N_{sim}} \sum_{q=1}^{3} \sum_{sim=1}^{N_{sim}} Y_{k,q,sim}(x)$$
(39)

$$\sigma_{k}(x) = \sqrt{\frac{1}{3 \cdot (N_{sim} - 1)}} \sum_{q=1}^{3} \sum_{sim=1}^{N_{sim}} (Y_{k,q,sim}(x) - y_{k}(x))^{2}$$
(40)

The mean and coefficient of variation of the vulnerability function at excitation x for the asset class is given by

$$y(x) = \sum_{k=1}^{7} w_k \cdot y_k(x)$$
⁽⁴¹⁾

$$v(x) = \frac{1}{y(x)} \sqrt{\sum_{k=1}^{7} w_k} \cdot \left(\sigma_k^2(x) + \left(y_k(x) - y(x)\right)^2\right)$$
(42)

where

 $k = \text{index to index buildings } k \in \{1, 2, \dots 7\}$

 $y_k(x)$ = mean damage factor for index building k at excitation x

y(x) = mean damage factor for the asset class at excitation x

 $\sigma_k(x)$ = standard deviation of damage factor for index building k at excitation x

v(x) = coefficient of variation of damage factor for the asset class at excitation x

 N_{sim} = number of simulations of structural response, component damage, and repair cost per index building per level of excitation x; we recommend N_{sim} between 20 and 100.

 w_k = moment-matching weight of index building k,

sim = index to Monte Carlo simulations, sim $\in \{1, 2, ..., N_{sim}\}$

 $Y_{k,sim}(x)$ = damage factor for index building k in simulation sim at excitation x.

With y(x), v(x), and an assumed parametric distribution of loss conditioned on x (typically either lognormal or beta bounded by zero and 1), one can estimate the probability of exceeding any specified loss or the loss

associated with any specified probability. If desired, higher central moments of vulnerability can be estimated as follows:

$$E[Y^{n}] = \sum_{k=1}^{7} \frac{w_{k}}{N_{sim}} \cdot \sum_{sim=1}^{N_{sim}} (Y_{k,sim}(x) - y(x))^{n}$$
(43)

where Y denotes uncertain damage factor for a specimen of the class of interest, and *n* is any moment of interest $n \in \{2, 3,\}$. For example, n = 3 refers to skewness of the distribution of damage factor conditioned on a value of IM and n = 4 refers to kurtosis.

4 Illustrative Examples

4.1 Example with one index building

Consider the following building class: 1-3 story reinforced concrete shearwall office buildings in a moderateseismicity region of the United States, designed and built since the adoption of the 2000 International Building Code.

4.1.1 Asset definition

Per step 1 in Section 3.2, the analyst begins by defining a typical example building. We used a real building that was familiar to us: the Business School at the University of Colorado Boulder, whose design documents we acquired. Where the design documents lacked necessary data, especially unit costs, we used the RSMeans (2009) square-foot cost manual, in particular the model M.120.

Table 7. Index building definition

Asset class (e.g., material, LLRS, height category, occupancy)		
Structural material (if used, from GEM building	Concrete, reinforced (CR)	
taxonomy)		
Lateral load resisting system (if used, from GEM	Wall (LWAL)	
building taxonomy)		
Broad category (choose one)	Shearwall	
Height category (if used, from GEM building	1-3 stories (H:1,3)	
taxonomy)		
Occupancy (if used, from GEM building taxonomy)	Offices (COM3)	
Other attribute 1		
Other attribute 2		
Other attribute 3		
Index building quality (if using 3 index buildings; choose one)	Typical	
Index building name (if a particular real building is selected)	CU Business School	
Index building model (if using local per-square-meter cost	M.120 in RSMeans (2009)	
manual)		
Stories	2	
Story height (m)	4	
Building height (m)	8	
Design year	2009	
Construction year	2009	
Labor cost as a fraction of total labor + material in	0.5	
construction cost		
Local labor cost as a fraction of US labor cost	1.0	
Small amplitude fundamental period of vibration T, sec	0.2	
Gamma (default = 1.3)	1.3	
Median collapse capacity, \hat{S}_{CT} , g, IM = Sa(T, 5%), geomean	0.83	
Logarithmic standard deviation of collapse capacity (default	0.8	
0.8)		
Design base shear as fraction of building weight Cs	0.048	
Cost manual reference (if used)	RS Means (2009)	
Total building cost (currency per m ²)	\$1,364	
Total building floor area (m ²)	4,460	
Total building construction cost (currency, RCN)	\$6.08M million	
Fraction f_1 construction cost as fraction of RCN	0.52	

The RSMeans M.120 model had the top components shown in Table 8. RSMeans (2009) only categorizes components to the level of NISTIR 6389 (shown in the table), so we estimated the likely FEMA P-58 classes based on our knowledge of the building. The demand parameters are taken from the FEMA P-58 fragility database, and the costs per square meter are taken from RSMeans.

	Nonstructural components					Structural	
Rank ¹	1	2	3	4	5	6	1
Description ²	Terminal & package	Plumbing fixtures	Lighting & branch	Partitions	Interior Doors	Exterior windows	RC shearwalls
	units		wiring			(curtain walls)	
NISTIR 6389 class ID ³	D3050	D2010	D5020	C1010	C1020	B2020	B1040
FEMA P-58 class ID ⁴	D3052.011d	(rugged)	C3034.001	C1011.001d	C1020.001	B2022.035	B1044.043
Demand param (choose one) ⁵	PFA	PFA	PTD	PTD	PTD	PTD	PTD
Cost per m ²	\$196	\$143	\$126	\$74	\$47	\$42	\$86
Total cost/m ² these	\$715						
items							
Total cost/m ²	\$1,364						
f ₁ (frac RCN in	0.52						
inventory)							

Table 8. Ranking of components in decreasing order of contribution to construction cost

1 Component categories in decreasing order of construction cost

2 The description associated with the NISTIR 6389 class ID

3 The 5-character component category from NISTIR 6389 (NIST 1999)

4 Fragility input parameter: PFA = peak floor acceleration, PTD = peak transient drift ratio

To complete Table 9a-g, we copied the parameter values from the FEMA P-58 PACT database. In the case of interior doors, the PACT database does not offer a fragility function, so we developed one based on the following rationale. A reasonable choice for the damage state is "door jams; replace." A door will jam if it is subjected to a residual drift that exceeds the tolerance between the door and the door frame. So the door is sensitive to peak residual drift (PRD), not peak transient drift. But we prefer to use peak transient drift as the demand parameter. We therefore assume that PRD = $0.25 \cdot (PTD - 0.005)$, i.e., peak residual drift is 0.25 times peak transient drift in excess of yield, and that yield occurs at 0.5% drift. These figures are generally taken from unpublished work by Deierlein et al. for FEMA P-58, who estimated PRD as a function of PTD for 4 building types and 4 levels of PTD. It is also assumed that a single damage state occurs: door is jammed, and that door jamming occurs at PRD θ = 0.0047 (based on 3/8 inch residual drift over a door height of 80 inches), with β = 0.4. For convenience, θ_{PTD} is taken as 0.024. Thus, θ_{PRD} = 0.25 $\cdot (0.024 - 0.005) = 0.0047$, as desired.

Component Specification, Rank #1					
NISTIR Class	D3050	Comp. name	Terminal & package units		
FEMA P-58	D3052.011d	Unit		Ea	
Class					
Demand	PFA	Ref (default Pact	Pact 1.0		
param		1.0)			
Fragility function		n	Repair cost by damage state		
Damage state	Median capacity	Log std deviation	P ₅₀ (median cost) Log std deviation		
1	0.25	0.4	38031	1.05	
2					
3					
4					

Table 9a. Fragility functions and unit repair costs, nonstructural component rank 1

* This is a large log std dev because PACT uses MECE damage states. Parameters of repair cost are therefore calculated from PACT 1.0 using Equations (1) and (2)

Table 9b. Fragility functions and unit repair costs, nonstructural component rank 2

Component Specification, Rank #2					
NISTIR Class	D2010	Comp. name	Plur	nbing fixtures	
FEMA P-58	N/A – rugged	Unit	N,	/A—rugged	
Class					
Demand	N/A – rugged	Ref (default Pact	N/A—rugged		
param		1.0)			
	Fragility function	n	Repair cost by damage state		
Damage state	Median capacity	Log std deviation	P ₅₀ (median cost) Log std deviation		
1					
2					
3					
4					

Table 9c. Fragility functions and unit repair costs, nonstructural component rank 3

Component Specification, Rank #3					
NISTIR Class	D5020	Comp. name	Lighting & branch wiring		
FEMA P-58	C3034.001	Unit		Ea	
Class					
Demand	PFA	Ref (default Pact	Pact 1.0		
param		1.0)			
Fragility function		n	Repair cost by damage state		
Damage state	Median capacity	Log std deviation	P ₅₀ (median cost) Log std deviation		
1	0.6	0.4	483	0.637	
2					
3					
4					

Component Specification, Rank #4						
NISTIR Class	C1010	Comp. name	Partitions			
FEMA P-58	C1011.001d	Unit	1	00 lf = 30m		
Class						
Demand	PTD	Ref (default Pact		PACT 1.0		
param		1.0)				
Fragility function		Repair cost by damage state				
Damage state	Median capacity	Log std deviation	P ₅₀ (median cost) Log std deviation			
1	0.0035	0.7				
2	0.0093	0.45				
3						
4						

Table 9d. Fragility functions and unit repair costs, nonstructural component rank 4

Table 9e. Fragility functions and unit repair costs, nonstructural component rank 5

Component Specification, Rank #5						
NISTIR Class	C1020	Comp. name	Interior doors			
FEMA P-58	C1021.001	Unit		Each		
Class						
Demand	PTD	Ref (default Pact	Porter judgment			
param		1.0)				
Fragility function						
	Fragility function	n	Repair co	st by damage state		
Damage state	Fragility function Median capacity	n Log std deviation	Repair co P ₅₀ (median cost)	st by damage state Log std deviation		
Damage state	Fragility function Median capacity 0.024	n Log std deviation 0.4	Repair co P ₅₀ (median cost) 500	st by damage state Log std deviation 0.4		
Damage state 1 2	Fragility function Median capacity 0.024	n Log std deviation 0.4	Repair co P ₅₀ (median cost) 500	st by damage state Log std deviation 0.4		
Damage state 1 2 3	Fragility function Median capacity 0.024	n Log std deviation 0.4	Repair co P ₅₀ (median cost) 500	st by damage state Log std deviation 0.4		

Table 9f. Fragility functions and unit repair costs, nonstructural component rank 6

Component Specification, Rank #6 (typical quality if 7 index buildings)					
NISTIR Class	B2020	Comp. name	Exterior wir	ndows (curtain walls)	
FEMA P-58	B2022.035	Unit	Each	(4' by 8' panel)	
Class					
Demand	PTD	Ref (default Pact	PACT 1.0		
param		1.0)			
	Fragility function	n	Repair cost by damage state		
Damage state	Median capacity	Log std deviation	P ₅₀ (median cost) Log std deviation		
1	0.0181	0.25	2500	0.17	
2					
3					

Component Specification, Structural Rank #1 (typical quality if 7 index buildings)						
NISTIR Class	B1040	Comp. name	Reinforced Concrete Structural Elements			
FEMA P-58	B1044.043	Unit	Ea	ach (900 sf)		
Class						
Demand	PTD	Ref (default Pact	PACT 1.0			
param		1.0)				
	Fragility function		Repair cost by damage state			
Damage state	Median capacity	Log std deviation	P ₅₀ (median cost)	Log std deviation		
1	0.0076	0.35	89,254	0.08		
2	0.0134	0.45	151,740	0.15		
3						
4						

Table 9g. Fragility functions and unit repair costs, structural component rank 1

Our inventory is taken from the building drawings; see Table 10.

Table 10	. Component	inventory	by story
----------	-------------	-----------	----------

Rank:	1	2	3	4	5	6	7
Comp.	Terminal &	Plumbing	Lighting &	Partitions	Interior	Exterior	Shearwalls
name	package	fixtures	branch		doors	windows	
	units		wiring				
Unit	Ea	Ea	Ea	30m	Ea	Ea (4' x 8')	Ea (84 m ²)
Story			Quantit	y (total)			
1	2	70	60	5	30	80	12.2
2	0	70	60	5	30	80	12.2
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							

4.1.2 Story-level vulnerability function, without collapse

Step 2, per Section 3.3, is to estimate story-level vulnerability. Our calculations are performed in a spreadsheet, so for illustration purposes here, we show sample calculations for a single component at a single level of motion. Consider item 1, terminal and package units (2 packaged HVAC units at ground level), subjected to floor-level acceleration x = 0.2g. Recalling Table 8 and Table 9a,

 $N_D = 1$

 $\theta_{1} = 0.25$

 $\beta_1 = 0.4$

P₅₀ = 37600 b = 0.17

From Equations (4), (5), and (6), $\beta_m = 0.3$ (pushover analysis),

$$\beta_{1,1}' = \sqrt{\beta_{1,1}^2 + \beta_m^2} = \sqrt{0.4^2 + 0.3^2}$$

= 0.5
$$\theta_{i,d}' = \theta_{i,d} \cdot \exp(0.84 \cdot (\beta_{i,d}' - \beta_{i,d}))$$

= 0.25 \cdot exp(0.84 \cdot (0.5 - 0.4))
= 0.27

Now applying Equation (3),

$$p_{1,1}(0.2) = \Phi\left(\frac{\ln(s/\theta'_{1,1})}{\beta'_{i,d}}\right)$$
$$= \Phi\left(\frac{\ln(0.2/0.27)}{0.5}\right)$$
$$= 0.27$$

From Equation (8),

$$m_{i,d} = P_{50,i,d} \exp(0.5 \cdot b^2)$$

= \$37600 \cdot exp(0.5 \cdot 1.4²)
= \$100,184

And from Equation (7),

$$E[C_1 | S_{1,1} = 0.2] = N_i \cdot \sum_{d=1}^{N_D} p_{i,d}(s) \cdot m_{i,d}$$
$$= 2 \cdot (0.27 \cdot 100, 184)$$
$$= \$54,099$$

Without showing the work related to the other acceleration-sensitive component, lighting and branch wiring, we evaluated the acceleration-sensitive vulnerability at $S_1 = 0.2g$,

$$E[C_a | S_1 = 0.2g] = \sum_{i=1}^{n} E[C_i | S_1 = 0.2g]$$

= \$54,099 + \$265
= \$54,364

Illustrating for drift-sensitive components at drift = 0.001, and without showing the calculations that lead up to this,

$$E\left[C_{d} \left| S_{1} = 0.001 \right] = \sum_{i=1}^{n} E\left[C_{i} \left| S_{1} = 0.001 \right] \right]$$
$$= \$1020 + 0 + 0$$
$$= \$1020$$

4.1.3 Structural analysis

Step 3 is to perform a structural analysis. We illustrate this case with the simplified structural analysis described in Section 3.4.2. We calculate the period T, using option 1 in Section 3.4.2.

 $T = 0.0610 \cdot 8^{0.75}$

= 0.29 sec ≈ 0.3 sec.

x = Sa(0.3 sec, 5%), g, geomean

Consider Sa(0.3, 5%) = 0.4g

 $PGA = 0.4 \cdot Sa(0.3g, 5\%) = 0.16g$

Consider PFA at floor 1, z = 0:

$$\begin{split} s_{1a}(0.4g) &= PGA + \varphi(h) \cdot (\Gamma \cdot S_a(T, 5\%) - PGA) \\ &= 0.16g + 0 \cdot (1.3 \cdot 0.4g - 0.16g) \\ &= 0.16g \\ &\leq S_{\max} \end{split}$$

At upper floors we need

$$\varphi(h) = \frac{z^2}{Z^5} \cdot \frac{(70Z^3 - 40Z^2z + 5Zz^2 + 2z^3)}{37}$$
 shearwall building

Evaluating at z = 0, 3, and 6m,

Floor	Height,	Mode
	m	shape
1	0	0.000
2	4	0.348
R	8	1.000

Now consider collapse fragility. We begin by calculating the design, 5 percent damped, spectral response acceleration parameter at short periods S_{DS} of Equation (28).

 $S_S = 0.30g$ (near Boulder, CO)

 $F_a = 1.2$ (site class C)

$$S_{DS} = 2/3 \cdot F_a S_S = 0.24 \text{g}$$

Now C_s , the seismic response coefficient of Equation (27). The response modification coefficient R and importance factor Ie are taken from ASCE 7-10 Tables 12.1-1 (R for an ordinary reinforced concrete shearwall) and 1.5-2 (I_e for risk category II building, which this building is per ASCE 7-10 Table 1.5-1).

$$R = 4$$

/ = 1

$$C_{S} = S_{DS}/(R/I) = 0.06g$$

FEMA P-695 offers no estimate of collapse margin ratio (CMR) for a reinforced concrete shearwall building. NIST (2010) studied a number of 1- to 4-story reinforced concrete shearwall buildings and estimated their CMRs in the range of 0.8 to 3.0. We therefore selected a middle ground, CMR = 2.0, and SSF = 1.15, consistent with the recommendations in Section 3.4.2

Applying Equation (25), the median collapse capacity can be estimated as

$$\hat{S}_{CT} = C_s \cdot 1.5 \cdot R \cdot CMR \cdot SSF$$
$$= 0.06 \cdot 1.5 \cdot 4 \cdot 2.0 \cdot 1.15$$
$$= 0.828g$$

We can now evaluate collapse probability per Equation (24). At $S_a(0.3 \text{ sec}, 5\%) = 0.4g$,

$$P_{c}(0.4g) = \Phi\left(\frac{\ln\left(x/\hat{S}_{CT}\right)}{\beta_{TOT}}\right)$$
$$= \Phi\left(\frac{\ln\left(0.4/0.828\right)}{0.8}\right)$$
$$= 0.18$$

4.1.4 Building-level vulnerability function

We can now move to step 4. Illustrating Equation (29) at x = 0.4g,

$$y(0.4g) = P_{c}(0.4g) + (1 - P_{c}(0.4g)) \cdot \left(\frac{\frac{1}{f_{1}}\sum_{h=1}^{N} E[C|S_{h} = s_{h}(0.4g)]}{RCN}\right)$$
$$= 0.18 + (1 - 0.18) \cdot \left(\frac{\frac{1}{0.52}(17,430 + 317)}{6,080,000}\right)$$
$$= 0.19$$

4.1.5 Mean vulnerability function and uncertainty

For an analysis of a single index building, the mean vulnerability function for the class is the same as that of the index building, which we have just calculated. The uncertainty is calculated using Equation (35):

$$v(0.4g) = \frac{0.25}{\sqrt{y(0.4g)}} = \frac{0.25}{\sqrt{0.19}} = 0.58$$

Repeating these calculations at $x \in \{0.01, 0.02, ..., 3.0g\}$ results in the vulnerability functions shown in Figure 6. Note that collapse governs vulnerability in this case, owing to fairly low design capacity.



Figure 6. Seismic vulnerability of 1-3 story RCSW office building in region of 0.17 \leq S_{MS} <0.5g, 1 index building example

4.2 Example with three index buildings

The approach is the same as in Example 1, but with a poor and superior quality variant. For illustration purposes this example merely substitutes highly fragile, moderately fragile, and low-fragility FEMA P-58 equivalents for the NISTIR component categories, and increases or decreases median collapse capacity by a factor of 1.3 for superior or poor quality, respectively. Figure 7 shows the resulting vulnerability functions. Note that in this particular case, the typical case (same as in the previous example) is very similar to the mean, though we do not yet know how generally true this is. The coefficient of variation has the same trend as in the 1-index-building case but is somewhat higher, which shows that a supposedly more sophisticated model does not necessarily produce lower uncertainty.

		-			
Component Description	Poor	Typical	Superior		
Exterior Windows (Curtain Wall)	B2022.032	B2022.035	B2022.071		
Partitions	C1011.001a	C1011.001c	C1011.001b		
Interior Doors	C1021.001	C1021.001	C1021.001		
Plumbing Fixtures (rugged)					
Terminal & Package Units	D3052.011b	D3052.011d	D3052.013k		
Lighting & Branch Wiring	C3034.001	C3034.001	C3034.002		
RC shearwalls	B1044.073	B1044.013	B1044.041		
Other attributes that vary between index buildings					
	Poor	Typical	Superior		
Number of stories	3	2	1		
S _S , g	0.2	0.3	0.4		

 Table 11. FEMA P-58 component types used for poor, typical, and superior quality variants of lowrise RC shearwall

 office building





example

4.3 Example with seven index buildings

For an example presenting the derivation of vulnerability functions for a class of highrise office buildings whose lateral force resisting system is a reinforced concrete moment resisting frame, and which is designed according to the International Building Code after 2000 and located in seismic design category D ($S_{D1} \ge 0.2$ g), USA, see the companion document by Kazantzi et al. (2015). Therein, seven index buildings selected by moment-matching are analysed in detail using nonlinear dynamic analysis and detailed 2D models.

5 Conclusions

These guidelines present an analytical method to estimate the uncertain repair-cost and fatality-rate vulnerability of a building class. It was our goal to make the procedures flexible enough that one can use the state of the art in second-generation performance-based earthquake engineering, or simple enough that, if resources do not permit such rigor (which can takes days or weeks of model development and analysis), the analyst can still create a reasonable vulnerability function for the building class, potentially with only hours or days of effort. It was also our goal to make the procedures simple enough that anyone with a master's degree in structural engineering can perform the analysis.

At their most rigorous, the procedures present a component-based approach similar to second-generation performance-based earthquake engineering procedures (PBEE-2) developed since about 2000 in the United States. These procedures are exemplified by the so-called PEER methodology and systematized in the FEMA P-58 guidelines. PBEE-2 is applicable to single, particular buildings, not classes of buildings, so we have added a moment-matching approach to estimate the vulnerability for a class of buildings as the weighted mixture of vulnerability functions for a small number of carefully selected sample buildings, each of which is analysed by PBEE-2.

The moment-matching approach has the analyst determine which features of buildings in the class matter most to the vulnerability of the building, that is, which three features (generally having to do with strength and configuration) most strongly tend to make a particular specimen of the class more or less vulnerable than the average for the class. The analyst estimates the joint probability distribution of those features, for example by a survey using field observations or remote sensing. The analyst then selects seven specimens of the class whose features span the three dimensions that matter most. By moment matching (a generalization of Gaussian quadrature), one selects seven sets of values of those features—these represent seven index buildings—and seven weights to assign to those index buildings, so that the weighted moments of the features of the index buildings matches the first five moments of the joint distribution: the mean, variance, etc. The expected value of the vulnerability function for the class then represents a 5th order Taylor-series estimate of the class' vulnerability function if one were able to analyse an infinite number of specimens of the class whose features match the joint distribution. Moment matching is essentially a way to efficiently and rigorously propagate the important building-to-building uncertainties to estimate the uncertain behaviour of the class, rather than simply varying quantities that can most conveniently be varied. .

To achieve our objective of creating building-class vulnerability functions analyses where resources are too limited for PBEE-2, we offer simpler options for structural analysis than the dozens of nonlinear dynamic analyses of a multiple-degree-of-freedom structural model that PBEE-2 requires. One level of simplification is nonlinear pseudostatic (pushover) analysis. An even greater degree of simplification is to idealize the building as a single-degree-of-freedom cantilever column and calculate its story drifts and floor accelerations based on its spectral acceleration response, spectral displacement response, and an appropriate mode shape. We

offer three such idealizations: for a moment-frame system, we idealize the building as if it were a column with constant shear and flexural stiffness, where the shear stiffness is low compared to flexural stiffness. For a shearwall system, we idealize the building as if it were a column with constant shear and flexural stiffness, where the shear stiffness is high compared to flexural stiffness. Both of these cantilever columns have a mode shape that can be determined from first principles. For a mixed structural system, one can use a triangular mode shape. We offer best-estimate building period functions for various structural systems, to inform the estimation of spectral acceleration response and spectral displacement response. As a fourth option, the analyst can use any structural analysis technique that the analyst's project team deems professionally sound, as long as the team explains and justifies the choice to the consumers of the vulnerability functions.

As another simplification for creating building-class vulnerability functions with limited resources, one can estimate the vulnerability of each building considering only the six to eight building components that contribute most to construction cost, rather than requiring the analyst to quantify and estimate damage to every teacup and doorknob. The resulting vulnerability functions are then scaled up to account for the missing building inventory, based on the assumption that the missing components are just as vulnerable as the components that are included in the analysis.

A final mechanism for simplifying the uncertainty propagation is to consider only three index buildings, or even just one, albeit with less confidence in the mean estimate and uncertainty. Procedures are offered for each case.

We rely heavily on a component and fragility and consequence database offered in FEMA P-58. (The user can create or apply other fragility and consequence functions, and we point to relevant guidance on how to do that.) Since the database is US-centric, its repair costs may be incorrect for use in other countries. Since we want the procedures to be applicable anywhere in the world, we offer a simple method to adjust US-centric repair costs to account for local labor rates, based on the assumption that a half the US cost of the repair effort is material and the other half is labor. Economists' Law of One Price holds that the materials cost the same everywhere, but labor rates vary. (One can readily ship a bag of Portland cement around the world, so the law of one price holds that arbitrage opportunities will quickly equalize the price. Not so for a carpenter.)

A companion work by D'Ayala et al. (2015) offers an alternative approach to estimating the seismic vulnerability of a building class, based on a whole-building approach, as opposed to the component based approach used here. The companion work is deemed to be applicable only to low- and midrise buildings (perhaps 1 to 7 stories).

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APPENDIX A Porter, K. and I. Cho (2013). Characterizing a building class via key features and index buildings for class-level vulnerability functions

Characterizing a building class via key features and index buildings for class-level vulnerability functions

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ABSTRACT: A challenge to characterizing the uncertain future seismic performance of a class of buildings is how to represent its variability with rigor and a small sample of individual buildings. Second-generation performance-based earthquake engineering (PBEE-2, e.g., ATC 2012) provides insight into the seismic performance buildings with rigorous propagation of uncertainty, nonlinear time-history structural analysis, performance measured in terms of dollars, deaths, and downtime, and reasonable independence from expert opinion. But its asset definition is deterministic: it works on one building at a time. If one could make the asset definition probabilistic and have the distribution of its attributes represent that of a specified class of buildings, then this enhanced version of PBEE-2 would allow one to treat classes of buildings and model the behavior of buildings at the societal level, such as for catastrophe risk modeling. In this work we only treat the question of how can one model a few buildings and know the sample to be representative of the class. We offer a procedure to select a sample that spans the readily observable features that matter most to the class. One selects those features, performs a field survey to characterize their joint probability distribution, and then uses a procedure called moment matching (MM) to design a few sample (index) buildings with key features whose weighted values have the same first few moments as does the population of the survey. One can then analyze the index buildings by PBEE-2 and combine the results using the MM weights. With 7 index buildings, one can match the first 5 moments (mean, variance, etc.) of 3 key features (e.g., design base shear, number of stories, etc.). The procedure allows one to create a class-level vulnerability function with PBEE-2's rigor and reflect all of the most-important variability within the class. Admittedly 7 PBEE-2 analyses are not cheap, but they are practical and they sample over the most seismically important features for the class.

1 INTRODUCTION

Large-scale loss estimates such as at the state or national level can play a significant role in seismic risk management. For example, insurers make pricing, underwriting, and reinsurance decisions based on portfolio loss estimates. Governments make mitigation funding decisions based on the estimated costeffectiveness of a mitigation effort using categorybased vulnerability models. In large-scale loss estimation, constructing models that reliably reflect important features of the population of interest is an essential step. Prior approaches to class-based vulnerability functions often resort to either expert opinion (e.g. ATC 1985) or statistical analyses of empirical loss data (e.g. Wesson et al. 2004) to characterize the underlying building stock. Analytical approaches have also evolved to model behavior of individual buildings (e.g., Czarnecki 1973, Porter et al. 2001) and building classes (e.g., Scholl et al. 1982, Kircher et al. 1997, Rossetto & Elnashai 2005). Scholl et al. (1982) treat the class by modeling an average building of the class, and applying the resulting vulnerability function to the class. Kircher et al. (1997) treat variability within

the class by modeling the pushover curve of a typical sample of the class and then adding dispersion to the pushover to capture variability within the class. Rossetto and Elnashai (2005) model a single characteristic building and explicitly quantifying the effect on damage from random variations in structural material properties and member dimensions.

We propose to treat variability within the class by PBEE-2 analysis of a small number of samples of the class (a few index buildings), where the samples differ in three or so of their most important features, i.e., the features that are believed to cause the largest variation of seismic performance within the class, say design base shear or number of stories. The sampling strategy employs the moment matching (MM) technique (a kind of quadrature) to specify a few weighted values in the joint distribution of the key features to replicate the joint distribution observed in the population. One then designs and analyzes buildings with those values and combines the weighted results of individual buildings to estimate the behavior of the class. This extends PBEE-2 from being solely a single-building methodology to one that can treat entire building classes. Note the difference in Figure 1 from other



Iterate to quantify uncertainty, vulnerability

Figure 1. The PEER PBEE-2 framework with an added uncertainty at the beginning: uncertain asset given asset class.

depictions of the PEER methodology: the asset definition is now among the uncertainties to be quantified and propagated through the analysis.

Rosenblueth (1975) did something like this. Julier and Uhlman (2002) show how the number of samples can be made very small while still retaining the first several moments of the joint distribution. The term "moment matching" was proposed by Ching et al. (2009), who applied Julier and Uhlman's (2002) method to propagate uncertainty within a PBEE-2 analysis of a single building. Note also that the use of moment matching should be familiar to the reader who has studied the formulation of finite elements: we use quadrature—a special case of MM—to integrate strains across the element. The math is fancy, but long familiar to structural engineering.

We illustrate the methodology by considering California reinforced concrete (RC) buildings. First one identifies the key seismic features, at least the most important ones that are readily observable or capable of being estimated without examining structural drawings. We reviewed the well-regarding references listed in Table 1. The most commonly cited important features are shown in the top of the list. Some of these authors mention other features, but many were not observable or could not be readily inferred without examining structural drawings.

Not shown in Table 1 is design base shear, which is probably as important as any of these, and can be inferred from location and approximate year built. But for the sake of illustration, and to avoid the distraction of calculating design base shear, let us select the features shown: number of stories, vertical irregularity, and plan irregularity. The methodology is general enough that if we were to drop plan irregularity and add design base shear, it would make no difference to the basic methodology, which is the point of the present manuscript. In any case, the top three in Table 1 are the features that are believed by the FEMA 154 authors (the only source that directly quantifies the importance of these features to classes of buildings) to most strongly affect collapse probability in design-level shaking.

Table 1. Salient seismic features of RC buildings.

	ASCE 7 (2010)	Concrete Coalition (2011)	ACI 318 (2005)	FEMA 154 (ATC 2002)	FEMA 310 (1998)	FEMA 273 (1997)	EQ Eng. Handbook (2003)	Chopra & Goel (2000)
Stories	x	×	x	x	x	x	×	×
Vert irreg	×	×	×	×	×	×	×	
Plan irreg	×	×		×	×	×		
Num bays	×	×			×	×		
Pounding					×	×		
Openings	×		×	×	×	×		
Captive col		×	×			×	×	

One can look deeper than the features that matter most to the class: Porter et al. (2004) showed how one can explore the features that matter most to the performance of a particular building. In that example, it was the ground-motion time history and the fragility of the components, as opposed to uncertainties in the force-deformation behavior of the components, which are fairly well constrained in engineered construction. It suggested the importance of nonlinear dynamic analysis over nonlinear pseudostatic and careful consideration of the variety of building components in the building.

Why not use the more familiar Monte Carlo simulation or its fancier cousin, Latin Hypercube simulation? First, it can be very costly to create and analyze realistic structural models that differ in the important features such as vertical irregularity and number of stories. Second, as Ching et al. (2009) show, MM can achieve the same accuracy in a few samples as 100 s of MCS simulations.

Once one selects the features to be examined, the next step is to estimate their joint distribution. In our case, discussed more later, we performed a field survey of 265 buildings in four metropolitan areas of California, observing the values of the key features in each. The number 265 seems a large enough, though we do not offer guidance on what constitutes "enough." Before discussing the illustrative application more, let us turn to the general question of how to analyze the statistics to select sample buildings.

2 MOMENT MATCHING

2.1 Recap moment matching

In MM, the continuous probability density function (PDF) of a random variable X (called **X** PDF hereafter) is replaced by a discrete, approximately equivalent probability mass function (PMF) consisting of a number of weighted delta functions, whose weights and positions are determined such that the first few moments (e.g., mean, covariance, and higher joint moment) of both the discrete PMF and the **X** PDF are identical.

Actually the delta functions are not constrained to have weight between 0 and 1.0, unlike a PMF, although they do sum to 1.0. Julier and Uhlman 2002 refer to them as the sigma set, but we continue to refer to them here as a PMF for brevity. In essence, the MM technique produces point estimates to replace the continuous joint distribution, just as was Rosenblueth's (1975) objective.

As discussed by Ching et al. (2009), when we seek to estimate the expectation $E[\mathbf{Y}]$ (where $\mathbf{Y} = g(\mathbf{X})$ and \mathbf{X} and \mathbf{Y} are *n*-dimensional random variables), if we know the moments of \mathbf{X} PDF up to the *p*th-order, then we can exactly estimate the $E[\mathbf{Y}]$ with the same *p*th-order accuracy, and similarly the variance of \mathbf{Y} (denoted by *Var*[\mathbf{Y}]) with $\lfloor p/2 \rfloor$ th-order accuracy ($\lfloor \cdot \rfloor$ means the integer part of the number).

Also, from the standpoint of computation and accuracy, MM offers the advantages of being able to estimate third and higher moments of **Y**, in contrast to FOSM (Ching et al. 2009). And if we care about skewness, which we often do, FOSM simply doesn't capture all the necessary information. MM by contrast can capture as many moments as desired: with three points on the distribution of a continuous random variable, we can match 5 moments. More points, more moments. FOSM can't do that.

Let X^* have a discrete PMF consisting of delta functions. If the PDF of X^* has the same moments as XPDF up to the *p*th-order, $E[g(X^*)]$ estimates E[bfY]with *p*-th order accuracy. We can similarly estimate Var[Y], $E[Y^3]$, and so on. How to attain the delta functions that exactly match the first several moments of XPDF? As described by Ching et al. (2009), one solves the nonlinear system of equations:

$$\begin{bmatrix} \chi_1^1 & \chi_2^1 & \cdots & \chi_q^1 \\ \chi_1^2 & \chi_2^2 & & \chi_q^2 \\ \vdots & & \ddots & \vdots \\ \chi_1^p & \chi_2^p & \cdots & \chi_q^p \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_q \end{bmatrix} = \begin{bmatrix} E[X^1] \\ E[X^2] \\ \vdots \\ E[X^p] \end{bmatrix}$$
(1)

$$\sum_{i=1}^{q} w_i = 1 \tag{2}$$

We have p + 1 equations and 2q unknowns, i.e., q delta functions each represented by location χ_i and weight w_i . Using q delta functions, we can exactly match the first p = (2q - 1) moments of the X PDF. Ching et al. (2009) suggested three ways to attain the delta functions to match the first p = (2q - 1) moments: (1) solve the nonlinear system of equations in Equations (1-2); (2) adopt the Gauss-quadrature integration points and weights, when the X PDF is in the similar form of a standard one in reference to Hildebrand (1956) or Abramowitz and Stegun (1972); (3) assume the locations a priori (referring to the most similar standard PDF) and then find the weights by solving the now linear system of equations in Equation (1). These recommendations prove highly valuable, but their work focused on some well-behaved cases where standard Gauss-quadrature can be exploited, lacking general cases and detailed investigations about possible problematic issues.

In what follows we present two practical and versatile approaches that meets our primary goal, i.e., robust construction of a real population of sample buildings to represent the building class: (1) set of separate 1D MM followed by an assembling procedure, and (2) multidimensional MM. Both methods involve directly solving the nonlinear system of equations. In each approach, we address implementation, problematic behaviors, and their remedies.

2.2 1D MM, solving nonlinear system of equations

We found in our building survey that many building attributes are not distributed like a standard parametric distribution (Gaussian, lognormal, uniform, etc.). Hence for our purposes it is important to establish a universal strategy to find the X^* PMF that accommodates general distributions. We present a numerical strategy. As an illustrative example, we show how to obtain three delta functions with 6 unknowns (i.e. 3 locations and 3 weights), which exactly match the first five moments of the original data. For generality, we solve the nonlinear system of equations using multivariate Newton-Raphson (NR) iteration. *Matlab* code is provided in the Appendix.

For a 3-dimensional variable (here, three attributes such as stories, vertical irregularity, plan irregularity), the problem is to find 6 unknowns

$$\chi = \{\chi_1, \chi_2, \chi_3, w_1, w_2, w_3\}$$
(3)

that obey the six conditions, i.e., normality (weights w sum to 1.0) and the first five moments match the original data. For systematic operation, we introduce a multivariate function

$$\mathbf{F}(\boldsymbol{\chi}) = \{F_1, F_2, F_3, F_4, F_5, F_6\}$$
(4)

that contains the full context given in Equation (1). Then, let $F(\chi) = 0$ which means for each entity

$$F_{j} = \sum_{i=1}^{3} w_{i} \chi_{i}^{j-1} - E(X^{j-1}) = 0 \quad j \in [1,6]$$
(5)

The problem now becomes a task of root finding. We perform multivariate NR iteration until convergence. The key flow can be summarized thus. With the initial guess

$$\boldsymbol{\chi}^{(0)} = \{\chi_1, \chi_2, \chi_3, w_1, w_2, w_3\}^{(0)}$$
(6)

we solve and update as

$$\mathbf{J} \, d\mathbf{\chi}^{(k)} = \mathbf{F}(\mathbf{\chi}^{(k-1)}) \tag{7}$$

$$\boldsymbol{\chi}^{(k)} = \boldsymbol{\chi}^{(k-1)} - d\boldsymbol{\chi}^{(k)} \tag{8}$$

where the Jacobian matrix has entities such that

$$J_{i,j} = \partial F_i / \partial \chi_j \quad \text{for } i \in [1,6], \ j \in [1,3]$$
(9)

$$J_{i,j} = \partial F_i / \partial w_j \quad \text{for } i \in [1,6], \ j \in [4,6]$$
(10)

The initial guess $\chi^{(0)}$ can significantly affect convergence. We recommend beginning with the initial locations around E[X] and $E[X] \pm (E[X^2])^{0.5}$, i.e., the expected value of each parameter and the expected value ± 1 standard deviation. When we tried other points, the multivariate NR iteration often diverged. From a large number of tests, the initial values for the weights appears to have relatively weak influence on the convergence, which is to some extent expected due to the fact that only the first power of the weight term appears in Equation (5), thus contributing linearly.

As exploratory examples, we applied 1D MM using the multivariate NR method to the vertical irregularity, plan irregularity, and building height of 265 California RC buildings. Given a good initial guess, all iterations converged in less than five steps, as one might expect given the well-known quadratic convergence rate of the NR method.

Table 2 presents basic statistics of the 265 RC buildings in 4 California cities: Pasadena, Los Angeles, San Jose, and San Francisco. In the table, N stands for total sample points, which slightly varies (some quantities could not be observed). The first five moments $E[X] \sim E[X^5]$ are given in Table 1. To quantify plan irregularity, we used a ratio (>15%) of both plan projections of the building beyond a reentrant corner to the plan dimension (i.e. following the reentrant corner irregularity definition of ASCE 7-10, Table 12.3-1), and for vertical irregularity we used a ratio of the first story height to that of upper stories (i.e. according to the vertical structural irregularity definition of ASCE 7-10, Table 12.3-2). Initial locations of the delta functions are also provided. As discussed before, a slight change of the initial locations can cause a significant divergence problem due to the contribution of the higher power of the location terms, as reflected from Equation (5).

Figure 2 presents a histogram and cumulative distribution function of the vertical irregularity data. The associated three delta functions are given in Figure 3. Note again that the delta functions are attained without resorting to standard parametric probability distributions, yet with error less than 1.0×10^{-15}

Table 2. Statistics of sample buildings.

	N	E[X]	$E[X^2]$	$E[X^3]$
Vertical Irregularity	259	1.474	2.326	3.961
Plan Irregularity	265	1.236	2.276	10.675
Number of stories	265	9.008	112.909	1930.499
		$E[X^4]$	$E[X^5]$	Initial χ_i
Vertical Irregularity		7.31	14.6	[1, 2, 3]
Plan Irregularity		95.26	1002	[1.4, 4.4, 1.1]
Number of stories		44091	1253122	[8, 112, 200]



Figure 2. Vertical irregularity in sample buildings



Figure 3. Delta functions to replace the observed distribution of vertical irregularity. Numbers show position and weight, respectively.

2.3 One-step multidimensional MM

In the preceding section, we showed a simple numerical approach to obtain three delta functions that exactly match the first five moments of each real quantity. Let us turn now to the three-dimensional (3D) case, such as number stories, degree of vertical irregularity, and degree of plan irregularity.

If **X** is a 3D random variable, we need nine delta functions, i.e., three delta functions for the PDF in each dimension. (We will later constrain the central delta

function of each axis to share a common location.) Then, we have 18 unknowns,

$$\chi = \{\chi_i, w_i\}, i \in [1,9]$$
(11)

along with 18 conditions: the normality of the weights, first five moments in the three dimensions

$$E(X_i^k) \ k \in [1,5] \text{ and } i \in [1,3]$$
 (12)

and the two equality conditions among three weights. Again, with a multivariate function

$$\mathbf{F}(\boldsymbol{\chi}) = \{F_1, \cdots, F_{18}\}$$
(13)

These conditions can be rewritten

$$F_1 = \sum_{i=1}^9 w_i - 1.0 \tag{14}$$

$$F_{(3k+i-2)} = w_{3i-2}\chi_{3i-2}^{k} + 5w_{3i-1}\chi_{3i-1}^{k} + w_{3i}\chi_{3i}^{k} - E(X_{i}^{k})$$
(15)

$$F_{17} = w_2 - w_5 \tag{16}$$

 $F_{18} = w_5 - w_8 \tag{17}$

In Equation 15, $k \in [1, 2, ..., 5]$ and $i \in [1, 2, 3]$. The last two equality conditions are added to make the three independent axes intersect at their middle points. Again, with these expressions, the problem becomes a simple root finding task of $\mathbf{F}(\boldsymbol{\chi}) = 0$. Thus, we perform the multivariate NR iteration until convergence. We expand the key flow of the 1D MM case, and in this 3D case the system size increases linearly with the number of dimensions, i.e., threefold, *not* to the power of the number of dimensions.

We can obtain nine delta functions notably by only one-step solving of the nonlinear system of equations, not by separate solution in each dimension. The solution after five iterations reaches the desired results of $2 \sim 3\%$ mean error. See Figure 4 and Table 3. In the table, the *i* = 1, 2, and 3 denote positions for plan irregularity, 4, 5, and are for vertical irregularity, and 7, 8, and 9 are for number of stories. The table shows that some of the positions converged to negative values; we adjusted them to as shown in bold, with minimal increase in error.

This one-step solution appears attractive, but exhibited several adverse behaviors. First, because of the large size of the system, it becomes difficult to suppress divergence in the iteration method. As shown in Figure 4, up to the fifth iteration we observer the typical quadratic convergence of multivariate NR iteration. However, as the iteration continues, the solution diverges exponentially.

In particular, considering the fifth power of the location terms in Equation (5), a slight change of location appears to severely disturb the stability of the equilibrium satisfied at the fifth iteration. Once the instability emerges, the error grows exponentially. Thus, we found it necessary to interfere. When



Figure 4. Convergence of multivariate NR for 3D MM.

Table 3. Nine delta functions of 3D MM after modification.

Before modification [mean error: 2.56%]			After modification [mean error: 2.64%]		
i	Xi	Wi	Xi	Wi	
1	0.197	0.329	0.197	0.329	
2	1.375	0.163	1.375	0.163	
3	2.324	0.134	2.324	0.134	
4	-5.222	-0.00056	1.097	-0.00056	
5	1.438	0.163	1.438	0.163	
6	11.910	0.004	11.910	0.004	
7	-10.183	0.001	4.487	0.001	
8	9.473	0.163	9.473	0.163	
9	30.871	0.041	30.871	0.041	

intuitively reasonable values for the delta functions emerge, we stopped the iteration and reached an acceptably accurate solution. Others may have to do the same.

Second, the one-step 3D MM method often produces non-positive location values in positively valued variables (or values that must exceed 1.0, as in our case of vertical irregularity) during the large-sized multivariate NR iteration. This behavior is commonly found in many tests using reliable tools such as *Matlab* or Mathematica. It is well known that negative weight values can emerge during the application of general MM technique to realistic problems, and the negative weights can be seen acceptable as addressed by many (e.g., Ching et al. 2009). However, the non-positive location has neither physical counterpart nor practical interpretation. How would one construct an index building with -10 stories? Hence, it is recommended to adjust negative locations and give them positive values. This modification of the final solution is justifiable since the associated weight values appear to be small compared to other weight values (e.g., see $w_4 = -0.00056$ and $w_7 = 0.001$ before modification in Table 3), and consequently tends to have minimal effect in terms of the mean error. Indeed, after modification of χ_4 (from -5.222 to 1.097) and χ_7 (from -10.183 to 4.487), the mean error increases by only 0.08% (i.e. from 2.56% to 2.64%).

We could have imposed additional constraints in the problem setup to obtain positive locations. We found that additional constraints are closed tied to the convergence problem and relatively large weights. Thus such an automatic solution is not recommended. We found it preferable to manually modify negative location terms, in part because of their small weights.

3 APPLICATION

3.1 Collecting data from field survey

We wished to establish a reliable library of probability distributions of salient observable seismic features. We conducted a field survey in several California cities. The cities were selected considering their high populations: Los Angeles (population of 4,018,080 as of the year 2000), San Jose (912,332), and San Francisco (739,426). In addition, the city of Pasadena (close to Los Angeles) was investigated as a proxy of smaller cities near a large one.

We focused on the downtown areas of the selected cities where old and new RC structures can be found, thus providing unbiased distribution of the key features. At this stage, we placed the emphasis on all types of RC buildings lower than \sim 40 stories in the chosen cities. We obtained 265 meaningful samples with distinguishable salient features as previously summarized in Table 2.

3.2 Design of index buildings using MM

Although the present application treats three features, i.e. vertical and plan irregularities and building height, for the purpose of generality we first describe the general procedure in *n*-dimensional space to construct index buildings using the MM technique. Throughout the explanation, we used three delta functions for every MM. Since the extension to include more delta functions is straightforward, we shall not dwell upon this issue.

In the present application, amongst the 3 approaches to selecting the delta functions, the most exact one is approach 2, that is creating a set of 1D MM points in each dimension, then assembling them in the final multidimensional space. This approach produces the highest accuracy, which is the principal reason for preferring it. In the previous section, we showed how the separate 1D MM method quickly and exactly matched the first several moments of original data, given a suitable initial guess.

Figure 5 illustrates the conceptual configuration of 3n delta functions (i.e., three delta functions for each independent, orthogonal axis) in the *n*-dimensional space. In the present practical problem, each axis represents one salient feature in the building class such as number of stories, degree of plan irregularity, and



Figure 5. *n*-dimensional MM with 3 delta functions in each dimension.

Table 4. Locations and weights in *n*-dimensional MM with three delta functions per dimension.

Index	X_1	 X_n	Weight
0	χ2	 $\chi_{2+3(n-1)}$	$1 - \Sigma w_{(3i-2)} - \Sigma w_{(3i)}$
1	χ1	 $\chi_{2+3(n-1)}$	w_1
2	χ3	 $\chi_{2+3(n-1)}$	<i>w</i> ₃
3	χ2	 $\chi_{2+3(n-1)}$	w_4
4	χ2	 $\chi_{2+3(n-1)}$	w_6
÷	÷	:	:
2 <i>n</i> −1	χ2	 $\chi_{1+3(n-1)}$	w_{3n-2}
2 <i>n</i>	χ2	 $\chi_{1+3(n-1)}$	W _{3n}

so on. Note that the intersection point shown at the center of Figure 5 has one identical weight value for all dimensions.

Thus, 2n + 1 index buildings are required to span the *n*-dimensional space, as shown in Table 4. We applied this methodology to construct samples to represent the target building class "California RC buildings," in which each feature is captured by separate 1D MM using three delta functions. The basic statistics of the class were given in Table 2. The final set of index buildings is presented in Table 5. That is, the table presents the locations and weights for 7 index buildings that substitute for the 265 buildings in our sample while preserving the first five moments of original data from 265 actual buildings.

Figure 6 shows the joint PDF of the two salient seismic features (number of stories and degree of vertical irregularity) along with 5 delta functions and associated weights. Table 6 gives the positions and weights of the delta functions shown in the figure; note again that a negative weight for index building 0 is acceptable. As shown in the figure, the joint PDF of two attributes from actual RC buildings can be irregular, potentially poorly described a smooth parametric distribution. However, MM can provide index buildings that precisely preserve the statistics of original population of RC buildings, thereby assuring the desired universal applicability.

Table 5. Features of 7 index buildings to match the 265 in the sample.

Index building	Vertical Irregularity	Plan irregularity	Stories	Weight
0	1.692	3.165	13	-1.043
1	1.138	3.165	13	0.487
2	2.695	3.165	13	0.051
3	1.692	1.097	13	0.956
4	1.692	10.960	13	0.006
5	1.692	3.165	4	0.521
6	1.692	3.165	35	0.020



Figure 6. Joint PDF of number of stories and degree of vertical irregularity in 265 California buildings (the surface) and 5 equivalent index buildings (delta functions) selected by MM.

Table 6. Index buildings to approximate the joint PDF of sto-ries and vertical irregularity in 265 California buildings.

Index building	Vertical irregularity	Stories	Weight
0	1.692	13	-0.08046
1	1.692	4	0.52194
2	1.692	35	0.02013
3	1.138	13	0.48708
4	2.695	13	0.05131

4 CONCLUSIONS

We have shown how one can characterize a building class with a small sample set of index buildings for purposes of creating a probabilistic definition of the assets in an asset class. Our immediate purpose was to create seismic vulnerability functions of asset classes for use in societal loss estimation, but there are probably other practical uses. For example, if one wishes to perform a PBEE-2 analysis of a building whose important features are not all perfectly known, the same procedure can be used to reflect that particular asset probabilistically. Essentially the goal was to treat the PBEE-2 asset definition as uncertain, rather than as a the deterministic, particular building PBEE-2 is currently formulated to address. The uncertainty must be rigorously quantified to ensure that the sample set accurately represents the class. The asset uncertainty can then be propagated through the PBEE-2 analysis just as uncertainty in hazard, structural response, damage, and loss are. By doing so, one can create an analytically derived seismic vulnerability model for a class of buildings where the most important uncertainties including building geometry, design base shear, etc. are rigorously quantified and propagated through the analysis.

We presented a universal methodology to construct sample sets to efficiently represent a building class. The methodology should cover any type of structure, as long as one can identify and measure the features that matter, and construct a (potentially joint) probability distribution of them.

The procedure begins by identifying the uncertain features that matter most. These may differ from the features that are most convenient for simulating, such as material properties or member dimensions. For practicality, we assumed that these most important by well-regarded prior works. Future research could test this assumption and more rigorously select the most important features. The top three features selected here are number of stories and degrees of vertical and plan irregularity. Further inclusion of other features would be straightforward; the methodology is readily expandable to additional or different dimensions.

Then one performs a field survey to determine the joint distribution of those features. We illustrated by performing a survey of 265 California RC buildings in 4 cities. To embrace arbitrary (nonparametric) PDFs of the key attributes, we provides two versatile tools for the MM: (1) create and then assemble a set of 1D MM samples; (2) a one-step multidimensional MM method. Both tools are addressed by solving a nonlinear system of equations. Approach 1 (separate 1D MM of each feature) appears to guarantee exact accuracy up to the several moments of original data. The multidimensional MM (approach 2) allows a fast onestep matching task, but with some error. Although both numerical MM methods have universal capability regardless of the PDFs, they exhibit strong sensitivity to initial conditions and convergence problems. Appropriate remedies are suggested herein.

As shown in the practical application, a small number of so-called index buildings proved sufficient to rigorously mimic the statistics of the original data. The small number is important because each index building must be analyzed using PBEE-2, which can be a timeconsuming process, especially in the construction of a structural model for each index building.

Since the methodology is capable of tackling arbitrary PDFs and also achieving both accuracy and computational efficiency, we can now proceed toward large-scale loss estimation with no restrictions on region, building class, and associated PDFs.

APPENDIX: MATLAB CODE FOR MM

% initial locations and weights of three delta functions x = [8.0; 112.0; 200.0; 0.2; 0.5; 0.3]; % defined by user

% perform Newton-Raphson (NR) iteration solution = multiNR(@targetFn, x, 20);

% definition of the multivariate NR iteration function x = multiNR(f, x, NumIterations) [y,dy]=f(x); for j = 1:NumIterations s = dy\y; x = x-s; [y, dy] = f(x); end

function [y, dy] = targetFn(x) % the first five moments defined by user % i.e., EX(1) = E(X), $EX(2) = E(X^2)$, etc.

 $\begin{array}{ll} EX(1) = 9.008; & EX(2) = 112.909; \\ EX(3) = 1930.499; & EX(4) = 44091.067; \\ EX(5) = 1253121.960; \\ n = length(x); \ y = zeros(size(x)); \ dy = zeros(n, n); \end{array}$

% initialization and definition of the target function y(1) = x(4) + x(5) + x(6) - 1.0;for i=2:6 $y(i)=x(4)*x(1)^{(i-1)}+x(5)*x(2)^{(i-1)}+x(6)*x(3)^{(i-1)}-EX(i-1);$ end

% Jacobian matrix [6, 6] dy(1,1:3) = 0.0; dy(1,4:6) = 1.0; for j=2:6; for i=1:3; dy(j,i) = x(i+3)*(j-1)*x(i)^(j-2); dy(j,i+3) = x(i)^(j-1); end; end

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APPENDIX B Seven-building asset definition tables; notes on simulation

B.1 Notes on number of stories, degree of vertical irregularity, and degree of vertical irregularity

For buildings on hillsides where the lowest story has varying heights above grade, above-grade stories include all those where at least half the story along at least one façade is above grade.

Degree of vertical irregularity is defined here as the maximum ratio of the height of story h to the height of story h + 1. Typically this is the ratio of the height of the ground story to the height of the story above it.

Degree of plan irregularity is defined here as follows:

- For L-shaped and T-shaped buildings: the smaller ratio of the plan width on one side of a plan setback (the wider wing) to the plan width on the other side of the setback (the narrower wing). By "smaller" is meant that the ratio is calculated in each of two orthogonal directions for each plan setback. The smaller of the two ratios is the degree of plan irregularity.
- For C, E, Z and other shapes: the larger ratio of the plan width on one side of a plan setback (the wider wing) to the plan width on the other side of the setback (the narrower wing). By "larger" is meant that the ratio is calculated in each of two orthogonal directions for each plan setback. The larger of the two ratios is the degree of plan irregularity.

B.2 Notes on Monte Carlo simulation

The analyst who performs Monte Carlo simulation and uses 7 index buildings is assumed here to possess sufficient skills so as not to require a detailed explanation of how to perform the simulation. The following notes are provided to clarify some issues that will arise during the simulation process.

Uncertainty in the asset definition. The 7 index buildings are deemed to capture the bulk of the uncertainty in the asset definition, i.e., the variability between buildings in an asset class. In addition, the within-building variability is deemed to be captured by the uncertain replacement cost new (RCN), the probabilistic depiction of the detailed (FEMA P-58) component categories, together with uncertainty in the hazard analysis, damage analysis, and loss analysis, as described next. By "probabilistic depiction of the detailed (FEMA P-58) component categories. By "probabilistic depiction of the detailed (FEMA P-58) component categories, together with uncertainty in the hazard analysis, damage analysis, and loss analysis, as described next. By "probabilistic depiction of the detailed (FEMA P-58) component categories." We refer to the fact that each NISTIR component category can be associated with up to 3 FEMA P-58 categories, generically labelled in the inventory tables as poor, typical, and superior quality, each with an associated probability of usage. Absent better information, take *RCN* as normally distributed with a coefficient of variation equal to 0.1.

Uncertainty in the hazard analysis. When performing a structural analysis to determine response conditioned on ground motion, do not include uncertainty in IM given magnitude, distance, and other

source, path, and site uncertainties. If the structural model is nonlinear pseudostatic, include uncertain structural response conditioned on IM. If the structural analysis is nonlinear dynamic, for each simulation, randomly select among the ground-motion time histories.

Uncertainty in the structural analysis. If the structural model is a nonlinear dynamic model, it is reasonable to assume that uncertainty in the ground-motion time history conditioned on IM and the uncertainty in the damage analysis swamp uncertainty in structural response given the ground-motion time history. While the analyst is free to include uncertainty in the structural model, this guideline does not require it. Those uncertainties can include member dimensions, material strength, elastic modulus, and elastic damping ratio.

Uncertainty in the damage analysis. The probability mass function of component damage state conditioned on structural response is as prescribed by Equation (3). We recommend simulating damage state for all components of a given type on a given story (for drift-sensitive components) or given floor (accelerationsensitive) as if their damage were perfectly correlated, that is, they all share the same damage state. Different components of different types on the same story or floor can be treated as independent, conditioned on structural response. Components on different stories or floors can also be treated as independent, conditioned on structural response.

Uncertainty in the loss analysis. The unit cost to repair a component from damage state *d* is deemed to be uncertain, and PACT suggests that some are normally distributed and some are lognormal. The coefficients of variation are often so small that there is no practical difference between the two. For simplicity it is assumed here that all unit costs are lognormally distributed, although the analyst is free to apply the probability distribution indicated in the PACT database. Their 10th, 50th, and 90th percentiles are denoted by p10, p50, and p90 in the PACT database. For each simulation, simulate the unit repair cost (the cost to repair a single component from a particular damage state) for each component type and damage state once and applying the same unit cost to all components of the same type and same damage state.

Correlation. Damage among similar components has an unknown degree of correlation. Until evidence to the contrary arises, the analyst can assume for the sake of simplicity that damage among specimens of the same component category on the same floor is perfectly correlated and all other correlations are zero. Unit repair costs among similar components has an unknown degree of correlation. Until evidence to the contrary arises, the analyst can assume for the sake of simplicity that unit repair costs among specimens of the same component category are perfectly correlated and all other correlations are zero.

B.3 Seven-building asset definition tables

Complete the following set of tables, one set for each index building.

Asset class (e.g., material, LLRS, height category, occupancy)	
Structural material (if used from GEM building	
taxonomy)	
Lateral load resisting system (if used, from GEM	
building taxonomy)	
Broad category (choose one)	Frame, shearwall, mixed
Height category (if used, from GEM building	
taxonomy)	
Occupancy (if used, from GEM building taxonomy)	
Other attribute 1	
Other attribute 2	
Other attribute 3	
Index building ID (choose one)	0, 1, 2, 3, 4, 5, 6
Index building weight (weights for IDs 0-6 sum to 1.0)	
Moments of top uncertainties	
Number of stories (mean and standard deviation)	Mean: Standard deviation:
Vertical irregularity (mean and standard deviation)	Mean: Standard deviation:
Plan irregularity (mean and standard deviation)	Mean: Standard deviation:
Index building attributes	
Number of stories	
Vertical irregularity (typically story 1 height/story 2	
height)	
Plan irregularity	
Index building name (if a particular real building is selected)	
Index building model (if using local per-square-meter cost	
manual)	
Story height (m)	
Building height (m)	
Design year	
Construction year	
Labor cost as a fraction of total labor + material in	
construction cost	
Local labor cost as a fraction of US labor cost	
Small amplitude fundamental period of vibration T, sec	
Gamma (default = 1.3)	
Median collapse capacity, S_{CT} , g, IM = Sa(T, 5%), geomean	
Logarithmic standard deviation of collapse capacity (default	
Design base shear as fraction of building weight C _s	
Cost manual reference (if used)	
Total building cost (currency per m ⁻)	
I otal building floor area (m)	
Firstian 6 inventors total as when the structure for the first	
Fraction J_1 inventory total construction cost as fraction of	
KUN	

Table 12. Index building definition (7 index buildings)

		Ν	onstructura	al compone	ents		Struc	tural
Rank ¹	1	2	3	4	5	6	1	2
Description ²								
NISTIR 6389 class ID ³								
Demand parameter								
(PFA or PTD) ⁴								
Cost per m ²								
Total cost/m ² these								
items								
Total cost/m ²								
f ₁ (frac RCN in								
inventory) ⁵								

Table 13. Ranking of components in decreasing order of contribution to construction cost (7 index buildings)

1 Component categories in decreasing order of construction cost

2 The description associated with the NISTIR 6389 class ID

3 The 5-character component category from NISTIR 6389 (NIST 1999)

4 Fragility input parameter: PFA = peak floor acceleration, PTD = peak transient drift ratio

5 fraction of total building replacement cost new that is included in these components. If considering nonstructural vulnerability only, fraction of nonstructural replacement cost new that is included in these components.

	Nonstructural Cor	mponent Specification	, Rank #1 (typical qual	ity variant) ¹
NISTIR Class ²		Comp. Name ³		
FEMA P-58		Unit⁵		
Class ⁴				
Demand	PFA or PTD	Reference ⁷		
param ⁶				
	Fragility function	n	Repair co	st by damage state
Damage state	Median ⁸	Beta ⁹	P_{50} (median) ¹⁰	Log standard deviation (b) ¹¹
1				
2				
3				
4			12	
		(Superior quality	variant) ¹²	
FEMA P-58		Unit		
Class				
Demand	PFA or PTD	Reference		
param				
P 0 · · · 0 · · · ·				
	Fragility function	n	Repair co	st by damage state
Damage state	Fragility function Median	n β	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state	Fragility function Median	n β	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state	Fragility functio Median	n β	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2 3	Fragility function Median	n β	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4	Fragility function Median	n β	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4	Fragility function Median	n β (Poor quality v	Repair co P ₅₀ (median) ariant) ¹³	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58	Fragility function Median	n β (Poor quality v Unit	Repair co. P ₅₀ (median) ariant) ¹³	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class	Fragility function Median	n β (Poor quality v Unit	Repair co. P ₅₀ (median) ariant) ¹³	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand	Fragility function Median PFA or PTD	n β (Poor quality v Unit Ref (default Pact	Repair co P ₅₀ (median) ariant) ¹³	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param	Fragility function Median PFA or PTD	n β (Poor quality v Unit Ref (default Pact 1.0)	Repair co. P ₅₀ (median) ariant) ¹³	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param	Fragility function Median PFA or PTD Fragility function	n β (Poor quality v Unit Ref (default Pact 1.0)	Repair co P ₅₀ (median) ariant) ¹³ Repair co	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state	Fragility function Median PFA or PTD Fragility function Median	n β (Poor quality v Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) ariant) ¹³ Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1	Fragility function Median PFA or PTD Fragility function Median	n β (Poor quality v Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) ariant) ¹³ Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1 2	Fragility function Median PFA or PTD Fragility function Median	n β (Poor quality v Unit Ref (default Pact 1.0) n Beta	Repair co. P ₅₀ (median) ariant) ¹³ Repair co. P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1 2 3	Fragility function Median PFA or PTD Fragility function Median	n β (Poor quality v Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) ariant) ¹³ Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)

Table 14a. Fragility functions and unit repair costs, nonstructural component rank 1 (7 index buildings)

1 A FEMA P-58 component class whose seismic installation conditions are typical of the index building; a subcategory of the NISTIR 6389 component class. Rank #1 refers to the fact that the generic NISTIR 6389 component contributes the most to the construction cost of the index building.

2 The 5-character component category from NISTIR 6389 (NIST 1999)

3 The description associated with the FEMA P-58 class

4 The alphanumeric code for the component type per FEMA P-58 (Applied Technology Council 2012)

5 Unit by which the component is measured for use in the fragility and consequence functions (e.g., each, 100m, etc.)

6 Fragility input parameter: PFA = peak floor acceleration, PTD = peak transient drift ratio

7 Reference to the fragility function and consequence function. Default = FEMA P-58 PACT software

8 Median = demand associated with 50% failure probability, in terms of the demand parameter

9 Beta = logarithmic standard deviation of capacity

10 P50 = median cost to repair one unit of the component from the specified damage state

11 Log standard deviation (b) = standard deviation of the natural logarithm of the cost to repair one unit of the component

12 Superior quality variant = a FEMA P-58 component class whose seismic installation conditions exemplify the subcategory of NISTIR 6389 category that one might find in a more seismically resistant specimen of the index building.

13 Poor quality variant = a FEMA P-58 component class whose seismic installation conditions exemplify the subcategory of NISTIR 6389 category that one might find in a less seismically resistant specimen of the index building.

	Nonstructural Co	mponent Specification	i, Rank #2 (typical qual	ity variant)	
NISTIR Class		Comp. Name			
FEMA P-58		Unit			
Class					
Demand	PFA or PTD	Ref (default Pact			
param		1.0)			
	Fragility function	n	Repair cos	st by damage state	
Damage state	Median	Beta	P ₅₀ (median)	Log standard deviation (b)	
1					
2					
3					
4					
		(Superior qualit	y variant)		
FEMA P-58		Unit			
Class					
Demand	PFA or PTD	Ref (default Pact			
param		1.0)			
	Fragility function	n	Repair cost by damage state		
Damage state	Median	Beta	P ₅₀ (median)	Log standard deviation (b)	
1					
2					
3					
4					
		(Poor quality	variant)		
FEMA P-58		Unit			
Class					
Demand	PFA or PTD	Ref (default Pact			
param		1.0)			
	Fragility function	n	Repair cos	st by damage state	
Damage state	Median	Beta	P ₅₀ (median)	Log standard deviation (b)	
1					
2					
3					
4					

Table 14b. Fragility functions and unit repair costs, nonstructural component rank 2 (7 index buildings)

	Nonstructural Co	mponent Specificatior	n, Rank #3 (typical qua	lity variant)
NISTIR Class		Comp. name		
FEMA P-58		Unit		
Class				
Demand	PFA or PTD	Ref (default Pact		
param		1.0)		
	Fragility function	n	Repair co	st by damage state
Damage state	Median	Beta	P ₅₀ (median)	Log standard deviation (b)
1				
2				
3				
4				
		(Superior qualit	y variant)	
FEMA P-58		Unit		
Class				
Demand	PFA or PTD	Ref (default Pact		
naram		1 0)		
param		1.07		
param	Fragility function	n	Repair co	st by damage state
Damage state	Fragility function Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state	Fragility function Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2	Fragility function Median	Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2 3	Fragility functio Median	Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4	Fragility function Median	Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4	Fragility function Median	n Beta (Poor quality	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58	Fragility function Median	n Beta (Poor quality Unit	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class	Fragility function Median	n Beta (Poor quality Unit	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand	Fragility function Median	n Beta (Poor quality Unit Ref (default Pact	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param	Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0)	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param	Fragility function Median PFA or PTD Fragility function	n Beta (Poor quality Unit Ref (default Pact 1.0)	Repair co P ₅₀ (median) variant) Repair co	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1 2	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1 2 3	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)

Table 14c. Fragility functions and unit repair costs, nonstructural component rank 3 (7 index buildings)

	Nonstructural Co	mponent Specification	n, Rank #4 (typical qua	lity variant)
NISTIR Class		Comp. name		
FEMA P-58		Unit		
Class				
Demand	PFA or PTD	Ref (default Pact		
param		1.0)		
	Fragility function	n	Repair co	st by damage state
Damage state	Median	Beta	P ₅₀ (median)	Log standard deviation (b)
1				
2				
3				
4				
		(Superior qualit	y variant)	
FEMA P-58		Unit		
Class				
Demand	PFA or PTD	Ref (default Pact		
param		1.0)		
		,		
	Fragility function	n	Repair co	st by damage state
Damage state	Fragility function Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state	Fragility function Median	Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2	Fragility function Median	Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2 3	Fragility function Median	Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4	Fragility function Median	Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4	Fragility function Median	Beta Beta (Poor quality v	Repair co P ₅₀ (median) 	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58	Fragility function Median	n Beta (Poor quality Unit	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class	Fragility function Median	n Beta (Poor quality v Unit	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand	Fragility function Median PFA or PTD	Peta Beta (Poor quality v Unit Ref (default Pact	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param	Fragility function Median PFA or PTD	n Beta (Poor quality Unit Ref (default Pact 1.0)	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param	Fragility function Median PFA or PTD Fragility function	n Beta (Poor quality v Unit Ref (default Pact 1.0)	Repair co P ₅₀ (median) variant) Repair co	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality v Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1 2	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality v Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1 2 3	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality v Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)

Table 14d. Fragility functions and unit repair costs, nonstructural component rank 4 (7 index buildings)

Nonstructural Component Specification, Rank #5 (typical quality variant)									
NISTIR Class		Comp. name							
FEMA P-58		Unit							
Class									
Demand	PFA or PTD	Ref (default Pact							
param		1.0)							
	Fragility function	n	Repair cost by damage state						
Damage state	Median	Beta	P ₅₀ (median)	Log standard deviation (b)					
1									
2									
3									
4									
		(Superior qualit	y variant)						
FEMA P-58		Unit							
Class									
Demand	PFA or PTD	Ref (default Pact							
param		1.0)							
p		2.07							
	Fragility function	n	Repair co	st by damage state					
Damage state	Fragility function Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state	Fragility function Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state 1 2	Fragility functio Median	Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state 1 2 3	Fragility function Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4	Fragility function Median	Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4	Fragility function Median	n Beta (Poor quality	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58	Fragility function Median	n Beta (Poor quality Unit	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class	Fragility functio Median	n Beta (Poor quality Unit	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand	Fragility function Median	n Beta (Poor quality Unit Ref (default Pact	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param	Fragility function Median PFA or PTD	n Beta (Poor quality Unit Ref (default Pact 1.0)	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param	Fragility function Median PFA or PTD Fragility function	n Beta (Poor quality Unit Ref (default Pact 1.0) n	Repair co P ₅₀ (median) variant) Repair co	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1	Fragility functio Median PFA or PTD Fragility functio Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1 2	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1 2 3	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)					

Table 14e. Fragility functions and unit repair costs, nonstructural component rank 5 (7 index buildings)

Nonstructural Component Specification, Rank #6 (typical quality variant)									
NISTIR Class		Comp. name							
FEMA P-58		Unit							
Class									
Demand	PFA or PTD	Ref (default Pact							
param		1.0)							
	Fragility function	n	Repair co	st by damage state					
Damage state	Median	Beta	P ₅₀ (median)	Log standard deviation (b)					
1									
2									
3									
4									
		(Superior qualit	y variant)						
FEMA P-58		Unit							
Class									
Demand	PFA or PTD	Ref (default Pact							
param		1.0)							
Fragility function									
	Fragility function	n	Repair co	st by damage state					
Damage state	Fragility function Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state	Fragility functio Median	Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state 1 2	Fragility function Median	Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state 1 2 3	Fragility function Median	Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4	Fragility function Median	Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4	Fragility function Median	n Beta (Poor quality	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58	Fragility function Median	n Beta (Poor quality Unit	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class	Fragility function Median	n Beta (Poor quality v Unit	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand	Fragility function Median	n Beta (Poor quality Unit Ref (default Pact	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param	Fragility function Median PFA or PTD	n Beta (Poor quality Unit Ref (default Pact 1.0)	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param	Fragility function Median PFA or PTD Fragility function	n Beta (Poor quality Unit Ref (default Pact 1.0)	Repair co P ₅₀ (median) variant) Repair co	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1 2	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1 2 3	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)					

Table 14f. Fragility functions and unit repair costs, nonstructural component rank 6 (7 index buildings)

	Structural Component Specification, Rank #1 (typical quality variant)									
NISTIR Class		Comp. name								
FEMA P-58		Unit								
Class										
Demand	PFA or PTD	Ref (default Pact								
param		1.0)								
	Fragility function	n	Repair cost by damage state							
Damage state	Median	Beta	P ₅₀ (median)	Log standard deviation (b)						
1										
2										
3										
4										
		(Superior qualit	y variant)							
FEMA P-58		Unit								
Class										
Demand	PFA or PTD	Ref (default Pact								
param		1.0)								
		,								
	Fragility function	n	Repair co	st by damage state						
Damage state	Fragility function Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)						
Damage state	Fragility function Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)						
Damage state 1 2	Fragility functio Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)						
Damage state 1 2 3	Fragility functio Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)						
Damage state 1 2 3 4	Fragility function Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)						
Damage state 1 2 3 4	Fragility functio Median	n Beta (Poor quality	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)						
Damage state 1 2 3 4 FEMA P-58	Fragility functio Median	n Beta (Poor quality Unit	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)						
Damage state 1 2 3 4 FEMA P-58 Class	Fragility function Median	n Beta (Poor quality Unit	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)						
Damage state 1 2 3 4 FEMA P-58 Class Demand	Fragility functio Median PFA or PTD	n Beta (Poor quality Unit Ref (default Pact	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)						
Damage state 1 2 3 4 FEMA P-58 Class Demand param	Fragility functio Median PFA or PTD	n Beta (Poor quality Unit Ref (default Pact 1.0)	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)						
Damage state 1 2 3 4 FEMA P-58 Class Demand param	Fragility function Median PFA or PTD Fragility function	n Beta (Poor quality Unit Ref (default Pact 1.0)	Repair co P ₅₀ (median) variant) Repair co	st by damage state Log standard deviation (b)						
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)						
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1	Fragility functio Median PFA or PTD Fragility functio Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)						
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1 2	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)						
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1 2 3	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)						

Table 14g. Fragility functions and unit repair costs, structural component rank 1 (7 index buildings)

Structural Component Specification, Rank #2 (typical quality variant)									
NISTIR Class		Comp. name							
FEMA P-58		Unit							
Class									
Demand	PFA or PTD	Ref (default Pact							
param		1.0)							
	Fragility function	n	Repair cost by damage state						
Damage state	Median	Beta	P ₅₀ (median) Log standard dev						
1									
2									
3									
4									
		(Superior qualit	y variant)						
FEMA P-58		Unit							
Class									
Demand	PFA or PTD	Ref (default Pact							
param		1.0)							
		=:•7							
	Fragility function	n	Repair co	st by damage state					
Damage state	Fragility function Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state	Fragility functio Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state	Fragility function Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state 1 2 3	Fragility function Median	n Beta	Repair co. P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4	Fragility function Median	n Beta	Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4	Fragility function Median	n Beta (Poor quality	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58	Fragility function Median	n Beta (Poor quality Unit	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class	Fragility function Median	n Beta (Poor quality Unit	Repair co. P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand	Fragility function Median PFA or PTD	n Beta (Poor quality Unit Ref (default Pact	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param	Fragility function Median PFA or PTD	n Beta (Poor quality Unit Ref (default Pact 1.0)	Repair co P ₅₀ (median) variant)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param	Fragility function Median PFA or PTD Fragility function	n Beta (Poor quality Unit Ref (default Pact 1.0) n	Repair co P ₅₀ (median) variant) Repair co	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1 2	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)					
Damage state 1 2 3 4 FEMA P-58 Class Demand param Damage state 1 2 3	Fragility function Median PFA or PTD Fragility function Median	n Beta (Poor quality Unit Ref (default Pact 1.0) n Beta	Repair co P ₅₀ (median) variant) Repair co P ₅₀ (median)	st by damage state Log standard deviation (b) st by damage state Log standard deviation (b)					

Table 14h. Fragility functions and unit repair costs, structural component rank 2 (7 index buildings)

			Nonstructura	l components	5		Structural		
Rank:	1	2	3	4	5	6	1	2	
Name									
Unit									
Story			Quantit	ty (total)					
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									
16									

Table 15. Component inventory by story

APPENDIX C ASCE 7-10 Table 12.2-1

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		ASCE 7 Section Where Response				Structural System Limitations Including Structural Height, h_n (ft) Limits ^c				
		Detailing Paguiraments	Response Modification	Overstrength	Deflection	Sei	ismic I	Design	ı Categ	gory
	Seismic Force-Resisting System	Are Specified	R ^a	Factor, Ω_0^g	Factor, $C_d^{\ b}$	В	С	\mathbf{D}^d	\mathbf{E}^{d}	\mathbf{F}^{e}
A.	BEARING WALL SYSTEMS									
1.	Special reinforced concrete shear walls ^{<i>l.m</i>}	14.2	5	21/2	5	NL	NL	160	160	100
2.	Ordinary reinforced concrete shear walls l	14.2	4	21/2	4	NL	NL	NP	NP	NP
3.	Detailed plain concrete shear walls ¹	14.2	2	21/2	2	NL	NP	NP	NP	NP
4.	Ordinary plain concrete shear walls ¹	14.2	11/2	21/2	11/2	NL	NP	NP	NP	NP
5.	Intermediate precast shear walls ¹	14.2	4	21/2	4	NL	NL	40^{k}	40^{k}	40^k
6.	Ordinary precast shear walls ¹	14.2	3	21/2	3	NL	NP	NP	NP	NP
7.	Special reinforced masonry shear walls	14.4	5	21/2	31/2	NL	NL	160	160	100
8.	Intermediate reinforced masonry shear walls	14.4	31/2	21/2	21/4	NL	NL	NP	NP	NP
9.	Ordinary reinforced masonry shear walls	14.4	2	21/2	13⁄4	NL	160	NP	NP	NP
10.	Detailed plain masonry shear walls	14.4	2	21/2	13⁄4	NL	NP	NP	NP	NP
11.	Ordinary plain masonry shear walls	14.4	11/2	21/2	11⁄4	NL	NP	NP	NP	NP
12.	Prestressed masonry shear walls	14.4	11/2	21/2	13⁄4	NL	NP	NP	NP	NP
13.	Ordinary reinforced AAC masonry shear walls	14.4	2	21/2	2	NL	35	NP	NP	NP
14.	Ordinary plain AAC masonry shear walls	14.4	11/2	21/2	11/2	NL	NP	NP	NP	NP
15.	Light-frame (wood) walls sheathed with wood structural panels rated for shear resistance or steel sheets	14.1 and 14.5	61/2	3	4	NL	NL	65	65	65
16.	Light-frame (cold-formed steel) walls sheathed with wood structural panels rated for shear resistance or steel sheets	14.1	61⁄2	3	4	NL	NL	65	65	65
17.	Light-frame walls with shear panels of all other materials	14.1 and 14.5	2	21/2	2	NL	NL	35	NP	NP
18.	Light-frame (cold-formed steel) wall systems using flat strap bracing	14.1	4	2	31/2	NL	NL	65	65	65
B.	BUILDING FRAME SYSTEMS									
1.	Steel eccentrically braced frames	14.1	8	2	4	NL	NL	160	160	100
2.	Steel special concentrically braced frames	14.1	6	2	5	NL	NL	160	160	100
3.	Steel ordinary concentrically braced frames	14.1	31/4	2	31/4	NL	NL	35 ^{<i>j</i>}	35 ^{<i>j</i>}	NP ⁱ

 Table 12.2-1
 Design Coefficients and Factors for Seismic Force-Resisting Systems

Continued

		ASCE 7 Section					Structural System Limitations Including Structural Height, h_n (ft) Limits ^c				
		Detailing Requirements	Modification Coefficient,	Overstrength	Deflection Amplification	Seismic Design			n Category		
	Seismic Force-Resisting System	Are Specified	\mathbf{R}^{a}	Factor, Ω_0^{g}	Factor, $C_d^{\ b}$	В	С	\mathbf{D}^d	\mathbf{E}^{d}	\mathbf{F}^{e}	
4.	Special reinforced concrete shear walls ^{<i>l</i>,<i>m</i>}	14.2	6	21/2	5	NL	NL	160	160	100	
5.	Ordinary reinforced concrete shear walls l	14.2	5	21/2	41/2	NL	NL	NP	NP	NP	
6.	Detailed plain concrete shear walls ¹	14.2 and 14.2.2.8	2	21/2	2	NL	NP	NP	NP	NP	
7.	Ordinary plain concrete shear walls ¹	14.2	11/2	21/2	11/2	NL	NP	NP	NP	NP	
8.	Intermediate precast shear walls ¹	14.2	5	21/2	41/2	NL	NL	40^{k}	40^{k}	40^k	
9.	Ordinary precast shear walls ¹	14.2	4	21/2	4	NL	NP	NP	NP	NP	
10.	Steel and concrete composite eccentrically braced frames	14.3	8	2 1/2	4	NL	NL	160	160	100	
11.	Steel and concrete composite special concentrically braced frames	14.3	5	2	41⁄2	NL	NL	160	160	100	
12.	Steel and concrete composite ordinary braced frames	14.3	3	2	3	NL	NL	NP	NP	NP	
13.	Steel and concrete composite plate shear walls	14.3	61/2	21/2	51/2	NL	NL	160	160	100	
14.	Steel and concrete composite special shear walls	14.3	6	21/2	5	NL	NL	160	160	100	
15.	Steel and concrete composite ordinary shear walls	14.3	5	21/2	41/2	NL	NL	NP	NP	NP	
16.	Special reinforced masonry shear walls	14.4	51/2	21/2	4	NL	NL	160	160	100	
17.	Intermediate reinforced masonry shear walls	14.4	4	21/2	4	NL	NL	NP	NP	NP	
18.	Ordinary reinforced masonry shear walls	14.4	2	21/2	2	NL	160	NP	NP	NP	
19.	Detailed plain masonry shear walls	14.4	2	21/2	2	NL	NP	NP	NP	NP	
20.	Ordinary plain masonry shear walls	14.4	11/2	21/2	11⁄4	NL	NP	NP	NP	NP	
21.	Prestressed masonry shear walls	14.4	11/2	21/2	13⁄4	NL	NP	NP	NP	NP	
22.	Light-frame (wood) walls sheathed with wood structural panels rated for shear resistance	14.5	7	21/2	41/2	NL	NL	65	65	65	
23.	Light-frame (cold-formed steel) walls sheathed with wood structural panels rated for shear resistance or steel sheets	14.1	7	21/2	41⁄2	NL	NL	65	65	65	
24.	Light-frame walls with shear panels of all other materials	14.1and 14.5	21/2	21/2	21/2	NL	NL	35	NP	NP	
25.	Steel buckling-restrained braced frames	14.1	8	21/2	5	NL	NL	160	160	100	
26.	Steel special plate shear walls	14.1	7	2	6	NL	NL	160	160	100	

ied)

		ASCE 7 Section Where Response				Structural System Limitations Including Structural Height, h_n (ft) Limits ^c				
		Detailing Requirements	Response Modification Coefficient	Overstrength	Deflection Amplification	Sei	smic I	smic Design C		gory
	Seismic Force-Resisting System	Are Specified	\mathbf{R}^{a}	Factor, Ω_0^{g}	Factor, $C_d^{\ b}$	В	С	\mathbf{D}^d	\mathbf{E}^{d}	\mathbf{F}^{e}
C.	MOMENT-RESISTING FRAME SYSTEMS									
1.	Steel special moment frames	14.1 and 12.2.5.5	8	3	51/2	NL	NL	NL	NL	NL
2.	Steel special truss moment frames	14.1	7	3	51/2	NL	NL	160	100	NP
3.	Steel intermediate moment frames	12.2.5.7 and 14.1	41/2	3	4	NL	NL	35 ^h	NP^h	NP^{h}
4.	Steel ordinary moment frames	12.2.5.6 and 14.1	31/2	3	3	NL	NL	NP ⁱ	NP ⁱ	NP ⁱ
5.	Special reinforced concrete moment frames ⁿ	12.2.5.5 and 14.2	8	3	51/2	NL	NL	NL	NL	NL
6.	Intermediate reinforced concrete moment frames	14.2	5	3	41/2	NL	NL	NP	NP	NP
7.	Ordinary reinforced concrete moment frames	14.2	3	3	21/2	NL	NP	NP	NP	NP
8.	Steel and concrete composite special moment frames	12.2.5.5 and 14.3	8	3	51/2	NL	NL	NL	NL	NL
9.	Steel and concrete composite intermediate moment frames	14.3	5	3	41/2	NL	NL	NP	NP	NP
10.	Steel and concrete composite partially restrained moment frames	14.3	6	3	51/2	160	160	100	NP	NP
11.	Steel and concrete composite ordinary moment frames	14.3	3	3	21/2	NL	NP	NP	NP	NP
12.	Cold-formed steel—special bolted moment frame ^{<i>p</i>}	14.1	31/2	3°	31/2	35	35	35	35	35
D.	DUAL SYSTEMS WITH SPECIAL MOMENT FRAMES CAPABLE OF RESISTING AT LEAST 25% OF PRESCRIBED SEISMIC FORCES	12.2.5.1								
1.	Steel eccentrically braced frames	14.1	8	21/2	4	NL	NL	NL	NL	NL
2.	Steel special concentrically braced frames	14.1	7	21/2	51/2	NL	NL	NL	NL	NL
3.	Special reinforced concrete shear walls l	14.2	7	21/2	51/2	NL	NL	NL	NL	NL
4.	Ordinary reinforced concrete shear walls ^{<i>l</i>}	14.2	6	21/2	5	NL	NL	NP	NP	NP
5.	Steel and concrete composite eccentrically braced frames	14.3	8	21/2	4	NL	NL	NL	NL	NL
6.	Steel and concrete composite special concentrically braced frames	14.3	6	21/2	5	NL	NL	NL	NL	NL

Table 12.2-1 (Continued)

Continued

		ASCE 7 Section				Structural System Limitations Including Structural Height, h_n (ft) Limits ^c				
		Where Detailing Requirements	Response Modification	Overstrength	Deflection	Seismic Design Cates				gory
	Seismic Force-Resisting System	Are Specified	R ^a	Factor, Ω_0^g	Factor, C_d^b	В	С	\mathbf{D}^d	\mathbf{E}^{d}	\mathbf{F}^{e}
7.	Steel and concrete composite plate shear walls	14.3	71⁄2	21/2	6	NL	NL	NL	NL	NL
8.	Steel and concrete composite special shear walls	14.3	7	21/2	6	NL	NL	NL	NL	NL
9.	Steel and concrete composite ordinary shear walls	14.3	6	21/2	5	NL	NL	NP	NP	NP
10.	Special reinforced masonry shear walls	14.4	51/2	3	5	NL	NL	NL	NL	NL
11.	Intermediate reinforced masonry shear walls	14.4	4	3	31/2	NL	NL	NP	NP	NP
12.	Steel buckling-restrained braced frames	14.1	8	21/2	5	NL	NL	NL	NL	NL
13.	Steel special plate shear walls	14.1	8	21/2	61⁄2	NL	NL	NL	NL	NL
E.	DUAL SYSTEMS WITH INTERMEDIATE MOMENT FRAMES CAPABLE OF RESISTING AT LEAST 25% OF PRESCRIBED SEISMIC FORCES	12.2.5.1								
1.	Steel special concentrically braced frames ^f	14.1	6	21/2	5	NL	NL	35	NP	NP
2.	Special reinforced concrete shear walls l	14.2	61/2	21/2	5	NL	NL	160	100	100
3.	Ordinary reinforced masonry shear walls	14.4	3	3	21/2	NL	160	NP	NP	NP
4.	Intermediate reinforced masonry shear walls	14.4	31/2	3	3	NL	NL	NP	NP	NP
5.	Steel and concrete composite special concentrically braced frames	14.3	51/2	21/2	41/2	NL	NL	160	100	NP
6.	Steel and concrete composite ordinary braced frames	14.3	31/2	21/2	3	NL	NL	NP	NP	NP
7.	Steel and concrete composite ordinary shear walls	14.3	5	3	41/2	NL	NL	NP	NP	NP
8.	Ordinary reinforced concrete shear walls ^{<i>l</i>}	14.2	51/2	21/2	41⁄2	NL	NL	NP	NP	NP
F.	SHEAR WALL-FRAME INTERACTIVE SYSTEM WITH ORDINARY REINFORCED CONCRETE MOMENT FRAMES AND ORDINARY REINFORCED CONCRETE SHEAR WALLS'	12.2.5.8 and 14.2	41⁄2	21/2	4	NL	NP	NP	NP	NP

 Table 12.2-1 (Continued)

		ASCE 7 Section	Response Modification Coefficient, R ^a		Deflection Amplification Factor, C_d^b	Structural System Limitations Including Structural Height, h_n (ft) Limits ^e					
		Where Detailing Requirements Are Specified		Overstrength Factor, Ω_0^{g}		Seismic Design Category					
	Seismic Force-Resisting System					В	С	\mathbf{D}^d	\mathbf{E}^{d}	\mathbf{F}^{e}	
G.	CANTILEVERED COLUMN SYSTEMS DETAILED TO CONFORM TO THE REQUIREMENTS FOR:	12.2.5.2									
1.	Steel special cantilever column systems	14.1	21/2	1¼	21/2	35	35	35	35	35	
2.	Steel ordinary cantilever column systems	14.1	11⁄4	11⁄4	11/4	35	35	NP ⁱ	NP ⁱ	NP ⁱ	
3.	Special reinforced concrete moment frames ⁿ	12.2.5.5 and 14.2	21/2	11⁄4	21/2	35	35	35	35	35	
4.	Intermediate reinforced concrete moment frames	14.2	11/2	11⁄4	11/2	35	35	NP	NP	NP	
5.	Ordinary reinforced concrete moment frames	14.2	1	11⁄4	1	35	NP	NP	NP	NP	
6.	Timber frames	14.5	11/2	11/2	11/2	35	35	35	NP	NP	
H.	STEEL SYSTEMS NOT SPECIFICALLY DETAILED FOR SEISMIC RESISTANCE, EXCLUDING CANTILEVER COLUMN SYSTEMS	14.1	3	3	3	NL	NL	NP	NP	NP	

Table 12.2-1 (Continued)

^aResponse modification coefficient, *R*, for use throughout the standard. Note *R* reduces forces to a strength level, not an allowable stress level.

^{*b*}Deflection amplification factor, C_d , for use in Sections 12.8.6, 12.8.7, and 12.9.2.

 $^{\circ}NL = Not Limited and NP = Not Permitted.$ For metric units use 30.5 m for 100 ft and use 48.8 m for 160 ft.

^dSee Section 12.2.5.4 for a description of seismic force-resisting systems limited to buildings with a structural height, h_n , of 240 ft (73.2 m) or less.

"See Section 12.2.5.4 for seismic force-resisting systems limited to buildings with a structural height, h_n , of 160 ft (48.8 m) or less.

^fOrdinary moment frame is permitted to be used in lieu of intermediate moment frame for Seismic Design Categories B or C.

^{*s*}Where the tabulated value of the overstrength factor, Ω_0 , is greater than or equal to 2½, Ω_0 is permitted to be reduced by subtracting the value of 1/2 for structures with flexible diaphragms.

^hSee Section 12.2.5.7 for limitations in structures assigned to Seismic Design Categories D, E, or F.

ⁱ See Section 12.2.5.6 for limitations in structures assigned to Seismic Design Categories D, E, or F.

³Steel ordinary concentrically braced frames are permitted in single-story buildings up to a structural height, h_n , of 60 ft (18.3 m) where the dead load of the roof does not exceed 20 psf

(0.96 kN/m²) and in penthouse structures.

^kAn increase in structural height, h_n , to 45 ft (13.7 m) is permitted for single story storage warehouse facilities.

In Section 2.2 of ACI 318. A shear wall is defined as a structural wall.

"In Section 2.2 of ACI 318. The definition of "special structural wall" includes precast and cast-in-place construction.

"In Section 2.2 of ACI 318. The definition of "special moment frame" includes precast and cast-in-place construction.

^oAlternately, the seismic load effect with overstrength, E_{min} , is permitted to be based on the expected strength determined in accordance with AISI S110. ^pCold-formed steel – special bolted moment frames shall be limited to one-story in height in accordance with AISI S110.

APPENDIX D Other commentary

D.1 Summary of class partitioning

This technique is an alternative to moment matching for selecting five or more index buildings to represent the variability of features in the population. It constitutes an actual partitioning of the population into a set of collectively exhaustive and mutually exclusive subclasses of buildings each of which is represented by a single index building. Similarly to moment-matching, the overall properties of the building population are simply approximated by the joint probability mass function (PMF) established by the index buildings. In simpler terms, this means that the distribution properties (e.g., mean and standard deviation of) the population are assessed by condensing the population to just the index buildings used, to each of which a certain weight is assigned, according to its actual membership (percentage of buildings it represents) in the entire population. For large statistical samples, formal clustering methods (e.g., k-means clustering) need to be used. For most simple cases, though, the intuition and knowledge of an analyst that is intimately familiar with his or her dataset will be enough to select a number of appropriate index buildings, again as characterized by *k* significant properties, and the appropriate probabilities of occurrence (or weights), *p_i*. Conceptually, this approach can be aided by loosely following these two steps:

1) Splitting: For each building characteristic, 2 or 3 subclasses are determined that partition the population to distinct parts, each of which represents at least 15% of the population's distribution for the specified characteristic

2) Merging: The total number of possible partitions for the entire population is between 2^k or 3^k subclasses (actually 2 to the number of dimensions split in two times 3 to the number of dimensions split in three parts). Starting from the smaller subclasses, any adjacent subclasses whose participation to the overall population is found to be less than, say, 5% should be concatenated and the sum of their participation percentage assigned to the new wider subclass. Subclasses with participation larger than 15% should not be merged with others. This process is terminated when the desired number of subclasses is reached, typically 7-12.

D.2 Approximate equivalence of building-to-building and within-building uncertainty

In Porter et al. (2002), building-specific seismic vulnerability functions were calculated for 19 variants of 4 woodframe index buildings, using assembly-based vulnerability (ABV, defined initially in Beck et al. 1999 and more clearly in Porter 2000). ABV is essentially the same as 2nd-generation performance based earthquake engineering as adopted by the Pacific Earthquake Engineering Research (PEER) Center and later in FEMA P-58, and now adapted to classes of building in the present guidelines. Porter et al. (2002) calculated the mean and coefficient of variation of damage factor for each index building at each of 20 levels of 5% damped elastic spectral acceleration response, and in one place plotted COV as a function of MDF. Figure 8 shows that relationship for nine variants of two index buildings, and a regression line for one variant of each index building. As others had previously found (W. Graf, personal communication 2002), and as shown in the figure, COV tends to be inversely related to MDF.

In later work, Porter (2010) showed that one can infer the uncertainty in the HAZUS-MH analytical vulnerability functions for its building classes (NIBS and FEMA 2012), along with the mean damage factor. Plotting COV versus MDF for 51 levels of IM and 128 building classes, one sees a similar inverse relationship between COV and MDF, as shown in Figure 9. What is relevant here is that the curves in Figure 8 illustrate within-building variability associated with record-to-record variability, uncertainty in structural model parameters, and uncertainty in component damage and unit repair costs. Figure 9 reflects total variability of the class-level vulnerability function, both within-building and building-to-building variability. The ratio of the curve in Figure 9 to either curve in Figure 8 sheds light on the contribution of between-building variability to total variability. Figure 10 extrapolates all three curves to MDF = 0.5 and plots the ratio of the within-building COV from the CUREE-Caltech index buildings to the total COV from the HAZUS-MH vulnerability functions. The ratio varies with MDF, but is generally around 1.4, which suggests that building-to-building variability is approximately equal to within-building variability, at least for building classes defined as broadly as those in HAZUS-MH. This is the ratio one would get if one were to estimate total uncertainty as the square root of the sum of the squares of within-building and building-to-building standard deviation of loss, and the two were equal.



Figure 8. COV versus mean damage factor for 10 variants of 2 index buildings from the CUREE-Caltech Woodframe Project (Porter et al. 2002).



Figure 9. COV versus mean damage factor for 128 HAZUS-MH building classes at each of 51 IM levels (Porter 2010)



Figure 10. Ratio of total COV to within-building COV as a function of MDF

D.3 Why rotate fragility functions about the 20th percentile

D.3.1 Problem: increasing uncertainty in a fragility function

Like many probabilistic seismic risk analysis procedures, GEM vulnerability guidelines make frequent use of lognormal cumulative distributions functions to idealize fragility functions. In Porter et al. (2013), lognormal cumulative distribution functions (CDFs) are used to idealize damage to building components and the collapse fragility of buildings. The parameters of a lognormal CDF are the median (denoted here by ϑ) and the logarithmic standard deviation (denoted here by θ). An example lognormal fragility function is shown in Figure 11.



Figure 11. Example lognormal fragility function

The fragility parameters are often conditioned on or used in conjunction with some modeling simplification. In the case of the damage analysis of a component at a particular story in a particular index building subjected to a particular base excitation, the simplification may be in the structural analysis, where we calculate the demand parameter to which the component is subjected. For example, we might calculate the peak transient drift ratio to which the component is subjected using a nonlinear pseudostatic structural analysis procedure. The base excitation might be measured in terms of the spectral acceleration response at some fixed period *T* and damping ratio *z*, and because of the structural analysis technique, the peak transient drift ratio in question has no associated uncertainty. The analyst knows that there would in fact be variability in drift conditioned on $S_a(T,z)$, because record-to-record variability among ground motions with the same $S_a(T,z)$ produces different drifts. How then to incorporate the added uncertainty?

D.3.2 Options

At least two options present themselves: do nothing, or increase the original β with an additional value to reflect modeling uncertainty. Kennedy and Short (1994), NIBS and FEMA (2009), and ATC (2012) all increase uncertainty at various stages of probabilistic seismic risk analysis. Let β_u denote the increase in the logarithmic standard deviation of capacity. Let β_d denote the logarithmic standard deviation of capacity.

The do-nothing alternative ($\theta_u = 0$) offers simplicity but opens the analysis to easy attacks on the basis of underestimated uncertainty. Including θ_u avoids attacks on the basis of underestimated uncertainty, can be done with a fairly simple calculation, and makes the analysis consistent with common practice as suggested above. In work for the GEM Vulnerability Consortium, the author considered these two options, their advantages and disadvantages, and selected the latter, that is, increasing uncertainty. However, doing so raises the question of exactly how to include the added θ .

The previously cited authors combined two logarithmic standard deviations together by calculating the square root of the sum of their squares (SRSS) and used that value as the total logarithmic standard

deviations (usually, anyway; a more complicated convolution is required to reflect a lognormally distributed excitation). This is justified by the consideration that if the quantity in question, the uncertain capacity of a building or component, is taken as the product of two uncertain, lognormally distributed, uncorrelated quantities, then their logarithmic variances properly sum and equivalently their logarithmic standard deviations are SRSSd. Let us proceed then to SRSS the logarithmic standard deviations as prior authors have done, as shown in Equation (44).

$$\beta_c = \sqrt{\beta_r^2 + \beta_u^2} \tag{44}$$

Kennedy and Short (1994) assert that keeping ϑ constant tends to bias the mean failure rate, denoted here by λ . However, they found that failure rate is insensitive to θ_c if one rotates about the 10th percentile. That is, λ is not greatly changed in one increases θ by Equation (45) and increases ϑ so that the value of excitation xassociated with failure probability of 0.10 is unchanged after increasing θ and ϑ . Let p denote the y-value of the rotation point, i.e., the point shared by the pre- and post-rotation fragility functions. Then ϑ' is given by Equation (45). The concept of rotating around p is illustrated in Figure 12.





Let us denote the failure rate after rotating about p by λ' , and let ϑ' denote the median capacity after rotating about p. Let G(x) denote the expected value of the rate at which earthquakes occur at a particular site and cause shaking of at least x, in events per year. (Let us leave it ambiguous whether that is the rate of mainshocks or the rate of all earthquakes.) Then λ and λ' are given by Equation (46) and (47) respectively.

$$\lambda = \int_{x=0}^{\infty} \Phi\left(\frac{\ln\left(x/\theta\right)}{\beta_d}\right) \left| \frac{dG(x)}{dx} \right| dx$$
(46)

$$\lambda' = \int_{s=0}^{\infty} \Phi\left(\frac{\ln\left(x/\theta'\right)}{\beta_c}\right) \left|\frac{dG(x)}{dx}\right| dx$$
(47)
D.3.3 Tornado diagrams for sensitivity testing

Let us begin the assessment of where to rotate with a series of sensitivity tests depicted in the form of tornado diagrams. See Howard (1988) for an early general discussion of tornado diagrams, or Porter et al. (2002) for an early discussion of tornado diagrams applied to seismic risk. Briefly, a tornado diagram depicts the sensitivity of a dependent variable to each of two or more uncertain independent variables. One chooses baseline values of the independent variables, along with high and low values of each independent variable. One can select quantitative definitions of baseline, high and low, such as mean or median, 90th, and 10th percentiles respectively, but it is not required. The dependent variable is assessed with all independent variables set to their baseline values. This is the baseline value for the dependent variable.

Then the first dependent variable is set to its high value and the dependent variable evaluated. The first dependent variable is next set to its low value and the dependent variable evaluated again. The difference between these two quantities is referred to as the swing for the 1st dependent variable. The first dependent variable is set back to its baseline value and the process repeated for the 2nd independent variable. The swing for the 2nd independent variable is evaluated. The process is repeated for each remaining independent variable: all are set to their baseline values and only one independent variable is varied at a time, i.e., not simultaneously.

The independent variables are sorted in decreasing swing and the results plotted with a horizontal bar chart, one bar for each independent variable. The independent variable with the highest swing is shown with the top bar, the 2nd highest swing with the second highest bar, etc. The dependent variable is measured on the horizontal axis. The ends of the bars are placed at the value of the dependent variable resulting from the low and high values of the independent variable in question. A vertical line is drawn through the baseline value of the dependent variable (i.e., with all the independent variables set at their baseline values).

The result is a bar chart that resembles a tornado in profile, hence the name. The bars at the top of the chart are the ones to which the dependent variable is most sensitive; the ones at the bottom, the least. One can use the tornado diagram to better decide which independent variable to investigate more closely, or to try to reduce its uncertainty, or other uses.

D.3.4 Applying tornado diagrams to the present problem

For present purposes, we want to see how sensitive mean failure rate λ is to the rotation point p, and just to be sure p is important, let us also check sensitivity to median capacity ϑ , the logarithmic standard deviations β_d and β_u , and the seismic hazard of the location where the asset is placed. The rationale is that if λ is more sensitive to ϑ or β than to p, there is less reason to worry about the best value of p. As with Kennedy and Short's (1994) example, let the dependent variable relate to the error in mean failure rate. Let us arbitrarily select 30 US locations with moderate to high seismic hazard, defined as a location where an ordinary building would have an ASCE 7-05 seismic design category B or above, i.e., $S_{D1} \ge 0.067g$. The locations are shown in Figure 13. The figure shows locations and a shaded map where the three shades from dark to light are high, moderate, and low seismicity, defined using the breakpoints $S_{D1} = 0.20g$ and 0.067g. (That is, high would be a count where at least one point has an ordinary building would have SDC D or above, moderate has at least

one location with SDC B or C, and low is SDC A.) They were selected to reflect a variety of hazard levels and tectonic environments (plate boundaries, continental interior, generally strike-slip, generally subduction).

Figure 13. Sample locations

Let us consider several error measures:

(1) Average absolute error, averaging error over 30 US locations with moderate to high seismic hazard. By "absolute error" is meant the difference between λ with rotation and λ without rotation. By "average" is meant that we average the difference over the n = 30 locations. That is,

$$\varepsilon_{aa} = \frac{1}{n} \sum_{i=1}^{n} \left(\lambda_i' - \lambda_i \right) \tag{48}$$

(2) Average relative error, same locations. By "relative error" is meant the absolute error divided by the λ without rotation. By "average" is meant the average of the relative error over the 30 locations.

$$\varepsilon_{ar} = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{\lambda_i' - \lambda_i}{\lambda_i} \right) \tag{49}$$

(3) Maximum absolute error, max over the 30 locations. By "maximum" is meant the largest of the 30 absolute values of the difference between λs with and without rotation. That is,

$$\varepsilon_{xa} = \max_{i} \left(\lambda_{i}' - \lambda_{i} \right) \tag{50}$$

(4) Minimum absolute error, min over 30 locations. Similar, but minimum. That is,

$$\varepsilon_{na} = \min_{i} \left(\lambda_{i}' - \lambda_{i} \right) \tag{51}$$

(5) Maximum relative error, max over 30 locations. Like 3, but normalized by λ without rotation.

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$$\varepsilon_{xr} = \max_{i} \left(\frac{\lambda_{i}' - \lambda_{i}}{\lambda_{i}} \right)$$
(52)

(6) Minimum relative error, min over 30 locations. Like 4, but normalized by λ without rotation.

$$\varepsilon_{nr} = \min_{i} \left(\frac{\lambda_{i}' - \lambda_{i}}{\lambda_{i}} \right)$$
(53)

(7) Relative error at the location of highest hazard. Highest hazard measured in terms of ASCE 7-05 S_{D1}.

$$\varepsilon_{hr} = \frac{\lambda_i' - \lambda_i}{\lambda_i} \qquad i : \max_i \left(S_{D1,i} \right)$$
(54)

(8) Relative error at the location of lowest hazard. Lowest hazard in terms of ASCE 7-05 S_{D1} .

$$\varepsilon_{lr} = \frac{\lambda_i' - \lambda_i}{\lambda_i} \qquad i: \min_i \left(S_{D1,i} \right) \tag{55}$$

Baseline, low, and high values are as shown in Table 1. Tornado diagrams for these 8 dependent variables follow the table.

	Trial values		
	Low	Baseline	High
median, ປ	0.2	0.5	1.0
logstdev without rotation, \mathcal{B}_d	0.2	0.4	0.6
logstdev to reflect uncertainty, $\boldsymbol{\beta}_u$	0.1	0.2	0.3
rotation point (percentile), p	0.1	0.2	0.5

Table 16. Sensitivity test parameter values









Figure 14. Tornado diagrams depicting results of sensitivity tests. The gray end reflects the lower value of the independent variable and the black end reflects the high value.

D.3.5 Observations from the tornado diagrams

First, the rotation point *p* is always the top, sometimes the second, most important variable, which implies that it is worth worrying about the selection of *p*. That is, *p* matters. It is the top variable for the average relative error (smallest $|\varepsilon_{ar}|$), which seem to be the most relevant one for a portfolio probabilistic seismic risk analysis. The second-highest variable is generally though not always θ .

Second, continuing with the assumption that we are primarily concerned about the average relative error ε_{ar} , p = 0.20 is the best choice of the three: the baseline value (p = 0.20) is the closest of the three values to producing $\varepsilon = 0.00$, i.e., it produces the least average bias. It is also the best choice for high-hazard sites (smallest $|\varepsilon_{hr}|$) though not for low-hazard sites (smallest $|\varepsilon_{hr}|$). For a range of values of ϑ , p = 0.20 is a good choice, generally producing average relative error $|\varepsilon_{ar}| \le 0.06$.

Three error measures favor p = 0.10: maximum absolute error and maximum relative error (smallest $|\varepsilon_{xa}|$ and smallest $|\varepsilon_{xr}|$), and relative error for the lowest hazard site (smallest $|\varepsilon_{lr}|$). Two favor p = 0.50: minimum absolute and minimum relative error (smallest $|\varepsilon_{na}|$ and smallest $|\varepsilon_{nr}|$). These five error measures are relevant if we are concerned with the minimizing error in extremes—the one site out of a portfolio with the highest or lowest hazard, or the one site where the rotation makes the biggest difference. Thus, for single-site risk analyses one might want to rotate about p = 0.10 or possibly 0.50. For general-purpose probabilistic seismic risk analysis however, these sensitivity tests suggest p = 0.20 is the best choice.

D.3.6 Honoring empirical fragility data

There is another consideration that has to do with honoring the data used to derive empirical fragility functions. Often one employs such fragility functions in probabilistic seismic risk analysis. Porter et al. (2010)

offer empirical seismic performance data regarding approximately 1,500 mechanical, electrical, and plumbing components shaken by earthquakes around the world between 1971 and 1999. The data come from the Electric Power Research Institute's (2003) eSQUG database. Porter et al. (2010) present the performance data in scatter diagrams where *x* denotes seismic excitation (peak ground acceleration) and *y* denotes failure probability. The scatter diagrams also include fragility functions derived from the data. The scatter diagrams show that the data tend to lie at *x* values where the *y*-value of the fragility function is between 0.0 and 0.20, i.e., at low failure rates.

This is intuitive because we like to design buildings and building components so that in real earthquakes they tend to have low failure rates. In any case, because the fragility functions are fit to the data, and because the data tend to lie at low failure probabilities, it is probably better to keep the fragility function fairly constant at those low failure probabilities. Adding uncertainty and rotating about the 50th percentile increases the fragility function at x-values below the median, meaning that it raises the fragility function across the range of x where the data lie. Rotating about low percentiles will tend to honor the data of empirically derived fragility functions better than rotating about the 50th percentile. There tends to be little difference between fragility functions rotated about the 10th and 20th percentiles, so the choice between the two does not matter much compared with choosing between them and the 50th.

D.3.7 Other considerations

As previously noted we tend to design buildings and other assets not to fail in common earthquakes. "Common earthquakes" here means those earthquakes that produce something near the average failure rate in some arbitrary asset, such as building collapses, where the averaging is over individual earthquakes. This means that when earthquakes do shake assets, failure rates are more commonly low than high. This is not always the case, but in general it tends to be true. Rotating about the 10th or 20th percentiles means that we will tend not to overestimate failure rates in common earthquakes; rotating about the 50th will tend to overestimate failure rates in common earthquakes, but be more accurate in rare earthquakes in which on the order of half or more assets fail.

D.3.8 Conclusions

It appears best to rotate fragility functions about the 20th percentile, especially in probabilistic seismic risk analysis of portfolios of assets. If one is more concerned with extrema, it can be better to rotate about the 10th or 50th. This conclusion is based on the foregoing sensitivity tests, which show the least bias in average error considering a variety of locations, plus considerations of honoring empirical fragility data and minimizing bias in common earthquakes.

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